Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads

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This article extends Leland’s results (1994) to examine effect of debt maturity on bond prices, credit spreads, the optimal amount of debt.
Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995) and Nielsen, Saa-Requejo, and Santa-Clara (1993) have examined the pricing of bonds with credit risk and arbitrary maturity.

- Examining debt values only, do not consider **optimal capital structure**.
- Assuming either that bankruptcy is triggered at an **exogenously specified asset value** or that is is triggered when **cash flow fails to cover interest payments**.
- Allowing **default-free interest rates to follow a stochastic process**.
This article derives closed-form results for the value of long-term risky debt (console bond) and yield spreads, for optimal capital structure.

The unleveraged value $V$ follows a diffusion process with constant volatility $\sigma$:

$$\frac{dV}{V} = [\mu(V, t)) - \delta] \, dt + \sigma \, dz,$$

(1)

where $\mu(V, t)$ is the total expected rate of return on asset value $V$; $\delta$ is the constant fraction of value paid out to security holders; and $dz$ is the increment of a standard Brownian motion. Note that $\delta = 0$ in Leland (1994) paper.
Leland (1994) (cont.)

- Any claim on the firm \( F(V, t) \) that continuously pays a nonnegative coupon, \( C \), per instant of time when the firm is solvent.

- It must satisfy the partial differential equation:

\[
(1/2)\sigma^2 V^2 F_{VV}(V, t) + (r - \delta)V F_V(V, t) - r F(V, t) + F_t(V, t) + C = 0.
\]

(2)

- When security have no explicit time dependence, the term \( F_t(V, t) = 0 \), Eq. (2) has the general solution

\[
F(V) = A_0 + A_1 V + A_2 V^{-X},
\]

(3)

where

\[
X = \frac{1}{\sigma^2} \left[ \left( r - \delta - \frac{\sigma^2}{2} \right) + \sqrt{\left( r - \delta - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right] .
\]

(4)

- Note that \( X = 2r/\sigma^2 \) as \( \delta = 0 \).
Leland (1994) (cont.)

- Boundary conditions for the debt is

\[
\begin{align*}
\text{At } V = V_B, \quad D(V) &= (1 - \alpha)V_B \\
\text{As } V \to \infty, \quad D(V) &\to C/r,
\end{align*}
\]

where a fraction \(0 \leq \alpha \leq 1\) of value will be lost to bankruptcy cost.

- We can use the above conditions to obtain the debt value by finding \(A_0, A_1, \text{ and } A_2:\)

\[
D(V) = C/r + [(1 - \alpha)V_B - C/r][V/V_B]^{-X}.
\]
Boundary conditions for the bankruptcy cost is

\[ At V = V_B, BC(V) = \alpha V_B \]  
\[ As V \rightarrow \infty, BC(V) \rightarrow 0. \]

We can use the above conditions to obtain the bankruptcy cost by finding \( A_0, A_1, \) and \( A_2: \)

\[ BC(V) = \alpha V_B (V/V_B)^{-X}. \]
Leland (1994) (cont.)

- Boundary conditions for the tax benefit is

\[
\begin{align*}
\text{At } V &= V_B, \ TB(V) = 0 \\
\text{As } V \to \infty, \ TB(V) \to \frac{\tau C}{r},
\end{align*}
\]  

(11) (12)

where \( \tau \) denotes the tax rate.

- We can use the above conditions to obtain the tax benefit by finding \( A_0, A_1, \) and \( A_2 \):

\[
TB(V) = \frac{\tau C}{r} - \left( \frac{\tau C}{r} \right) \left( \frac{V}{V_B} \right)^{-X}.
\]

(13)
Leland (1994) (cont.)

The total value of the firm $v(V)$ is

$$v(V) = V + TB(V) - BC(V)$$

$$= V + (\tau C/r) [1 - (V/V_B)^{-X}] - \alpha V_B (V/V_B)^{-X}.$$  (14)

where $\tau$ denotes the tax rate.

The equity value is given by

$$E(V) = v(V) - D(V)$$

$$= V - (1 - \tau)C/r + [(1 - \tau)C/r - V_B] [V/V_B]^{-X}.$$  (15)
Finite Maturity Debt

Consider a bond issue with maturity $t$ periods from the present, which continuously pays a constant coupon flow $c(t)$ and has principal $p(t)$.

Let $\rho(t)$ be the fraction of asset value $V_B$ which debt of maturity $t$ receives in the event of bankruptcy.

Using risk-neutral valuation, and letting $f(s; V, V_B)$ denote the density of the first passage time $s$ to $V_B$ from $V$ when the drift rate is $(r - \delta)$, gives debt with maturity $t$ the value

\[
d(V; V_B, t) = \int_0^t e^{-rs}c(t) [1 - F(s; V, V_B)] \, ds
+ e^{-rt}p(t) [1 - F(t; V, V_B)]
+ \int_0^t e^{-rs} \rho(t) V_B f(s; V, V_B) \, ds.
\]

(16)
Integrating the first term by parts yields

\[
d(V; V_B, t) = \frac{c(t)}{r} + e^{-rt} \left[ p(t) - \frac{c(t)}{r} \right] [1 - F(t)]
\]

\[
+ \left[ \rho(t)V_B - \frac{c(t)}{r} \right] G(t)
\]

where

\[
G(t) = \int_0^t e^{-rs} f(s; V, V_B) \, ds.
\]

We can find expressions for \( F(t) \) from Harrison (1990) and for \( G(t) \) from Rubinstein and Reiner (1991).
Finite Maturity Debt (cont.)

Note that as $t \to \infty$,

$$d(V; V_B, t) \to \frac{c(\infty)}{r} \left[ 1 - \left( \frac{V}{V_B} \right)^{-X} \right] + \rho(\infty) V_B \left( \frac{V}{V_B} \right)^{-X}$$

which is the same equation as Leland (1994) derived for infinite-horizon risky debt when $\rho(\infty) = (1 - \alpha)$. 

Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads
Finite Maturity Debt (cont.)

- As in Longstaff and Schwartz (1995), the analysis above has assumed that $V_B$ is an exogenous constant.

- For arbitrary capital structures, however, this is unlikely to be optimal.

- If the firm has only a single issue of debt outstanding, debt service requirements are time-dependent. Prior to maturity, asset value may be low (less than $P$) but still sufficient to justify paying the coupon to avoid default.

- A debt structure consistent with a constant endogenous-determined bankruptcy level $V_B$. This capital structure has time-independent debt service payments.
A Stationary Debt Structure

- The firm continuously sells a constant (principal) amount of new debt with maturity of $T$ years.
- New bond principal is issued at a rate $p = (P/T)$ per year, where $P$ is the total principal value of all outstanding bonds.
- As long as the firm remains solvent, at any time $s$, the total outstanding debt principal will be $P$, and have a uniform distribution of principal over maturities in the interval $(s, s + T)$.
- Bonds with principal $p$ pay a constant coupon rate $c = (C/T)$ per year, where $C$ is the total coupon paid by all outstanding bonds per year.
A Stationary Debt Structure (cont.)

• Assume that $\rho(t) = (1 - \alpha)/T$.

• Let $D(V; V_B, T)$ denote the total value of debt, when debt of maturity $T$ is issued.

$$D(V; V_B, T) = \int_0^T d(V; V_B, t)dt$$

$$= \frac{C}{r} + \left( P - \frac{C}{r} \right) \left( \frac{1 - e^{-rT}}{rT} - I(t) \right)$$

$$+ \left( (1 - \alpha)V_B - \frac{C}{r} \right) J(t), \quad (21)$$

where

$$I(T) = \frac{1}{T} \int_0^T e^{-rt} F(t)dt$$

$$J(T) = \frac{1}{T} \int_0^T G(t)dt.$$
A Stationary Debt Structure (cont.)

Following Leland (1994) (Eq. (14)), total firm value will be

\[ v(V; V_B) = V + \frac{\tau C}{r} \left[ 1 - \left( \frac{V}{V_B} \right)^{-X} \right] - \alpha V_B \left( \frac{V}{V_B} \right)^{-X}, \]  

(22)

where we recall in this paper the payout rate \( \delta \) is not zero.

The value of equity is given by

\[ E(V; V_B, T) = v(V; V_B) - D(V; V_B, T). \]  

(23)
Determining the Bankruptcy-Triggering Asset Level $V_B$

To determine the bankruptcy-triggering asset value $V_B$ endogenously, we solve the equation

$$\frac{\partial E(V; V_B, T)}{\partial V} \bigg|_{V=V_B} = 0.$$  \hspace{1cm} (24)

The solution for $V_B$ is

$$V_B = \frac{(C/r)((A/(rT) - B) - AP/(rT) - \tau CX/r}{1 + \alpha X - (1 - \alpha)B}.$$  \hspace{1cm} (25)

Observe that $V_B$ will depend upon the maturity of debt chosen, for any given $P$ and $C$. 

Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads
The smooth-pasting condition has the property of maximizing (with respect $V_B$) both the value of equity and the value of the firm, subject to $E(V) \geq 0$ for all $V \geq V_B$.

It also implies $E(V_B)_{VV} \geq 0$. 
The bankruptcy condition can be further analyzed by computing the expected appreciation of equity around the endogenous bankruptcy trigger.

Since $E_V = 0$ when $V = V_B$, the appreciation of equation simplifies to

$$dE \bigg|_{V=V_B} = \frac{1}{2} \sigma^2 V^2 E_{VV} \bigg|_{V=V_B} dt \geq 0.$$  \hspace{1cm} (26)
Determining the Bankruptcy-Triggering Asset Level $V_B$ (cont.)

- Evaluating and further simplifying yields

$$dE \big|_{V=V_B} = ((1 - \tau)C + p)dt - (d(V_B; V_B, T) + \delta V_B)dt,$$  \hspace{1cm} (27)

where $d(V_B; V_B, T) = (1 - \alpha)V_B/T$.

- The l.h.s of Eq. (27) is the change in the equity value at $V = V_B$; the r.h.s is the additional cash flow required from equity holders for current debt service.
  - The cost of debt service (the after-tax coupon expense plus the principal expense less the sum of the revenues from selling an equal principal amount of debt at their market price).
  - The cash flow available for payout generated by the firm’s activities.
For long term debt, $V_B < P$.

However, as $T \to 0$, $V_B \to P/(1 - \alpha)$, which exceeds $P$ when $\alpha > 0$.

- When debt is short term and $\alpha > 0$, bankruptcy will occur despite net worth being positive.
- Because the anticipated equity appreciation does not warrant the additional contribution required from equity holder to avoid default on bond service payment.
The tax deductibility is lost whenever $V$ falls below some value $V_T$, where $V_T > V_B$.

\[
V_B = \frac{(C/r)(A/(rT) - B) - AP/(rT)}{1 + X(\tau C/(rV_T) + \alpha) - (1 - \alpha)B},
\]

where $V_B$ in Eq. (28) will exceed $V_B$ in Eq. (25).

This lowers the value of debt and firm.
The “base” case: \( r = 0.075, \tau = 0.35, \alpha = 0.5, \) and \( \sigma = 0.2 \).

Assume that the firm pays out an amount to security holders equal to \( \delta V \), with \( \delta = 0.07 \).

Presume that the tax deductibility of debt is lost when all available cash flow is needed to pay interest to bond holders.

- At \( V = V_T \) such that \( \delta V_T = C \).
- When \( V_B < V < V_T \), current equity holders willingly contribute to the firm to avoid default (e.g., equity dilution).

At the initial asset value \( (V=100) \), we assume the coupon \( (c) \) is set so that newly-issued debt sells at par value \( (d = p) \).
Optimal Leverage

Figure 1: Firm value as a function of leverage.

- The leverage ratio which maximizes firm value is larger for debt with longer maturity.
- The maximize firm value is also greater.
## Optimal Leverage (cont.)

<table>
<thead>
<tr>
<th>Maturity (Year)</th>
<th>Firm Value (Dollars)</th>
<th>Bankruptcy Trigger</th>
<th>Optimal Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>104.10</td>
<td>27.70</td>
<td>19</td>
</tr>
<tr>
<td>1.0</td>
<td>104.85</td>
<td>28.80</td>
<td>22</td>
</tr>
<tr>
<td>2.0</td>
<td>106.00</td>
<td>30.55</td>
<td>26</td>
</tr>
<tr>
<td>5.0</td>
<td>108.25</td>
<td>35.75</td>
<td>37</td>
</tr>
<tr>
<td>10.0</td>
<td>110.45</td>
<td>36.60</td>
<td>43</td>
</tr>
<tr>
<td>20.0</td>
<td>111.95</td>
<td>35.30</td>
<td>46</td>
</tr>
<tr>
<td>Infinity</td>
<td>113.80</td>
<td>32.80</td>
<td>49</td>
</tr>
</tbody>
</table>

**Table 1:** Characteristic of Optimally Levered Firm

## Optimal Leverage Ratio

- Positive correlation between leverage and debt maturity (consistent with empirical evidence of Barclay and Smith (1995a)).
- For any maturity, the optimal leverage ratio will fall when $\sigma$ and $\alpha$ increase and $r$ decrease.
Optimal Leverage (cont.)

<table>
<thead>
<tr>
<th>Maturity (Year)</th>
<th>Firm Value (Dollars)</th>
<th>Bankruptcy Trigger</th>
<th>Credit Spread, Total Debt (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>104.10</td>
<td>27.70</td>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
<td>104.85</td>
<td>28.80</td>
<td>0</td>
</tr>
<tr>
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<td>30.55</td>
<td>0</td>
</tr>
<tr>
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<td>13</td>
</tr>
<tr>
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<td>36.60</td>
<td>55</td>
</tr>
<tr>
<td>20.0</td>
<td>111.95</td>
<td>35.30</td>
<td>86</td>
</tr>
<tr>
<td>Infinity</td>
<td>113.80</td>
<td>32.80</td>
<td>107</td>
</tr>
</tbody>
</table>

Table 1: Characteristic of Optimally Levered Firm

Credit Spread

- Credit Spreads at the optimal leverage are negligible for issuance maturities of 2 years or less.

- Longstaff and Schwartz (1995) report that, over the period 1977-1992 average credit spreads on Moody's Industrial bond ranged from 48 bps (Aaa) to 184 bps (Baa), with an average credit spread of 109 bps (investment-graded).
Optimal Leverage (cont.)

<table>
<thead>
<tr>
<th>Maturity (Year)</th>
<th>Firm Value (Dollars)</th>
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<td>32.80</td>
<td>49</td>
</tr>
</tbody>
</table>

Table 1: Characteristic of Optimally Levered Firm

Write-Down for Debt

- Write-Down in this model is \((1 - (1 - \alpha)\frac{V_B}{P})\)
  \((1 - ((1 - 0.5) \times 35.3)/(111.95 \times 0.46)) = 65.7\).

## Optimal Leverage (cont.)

<table>
<thead>
<tr>
<th>Maturity (Year)</th>
<th>Firm Value (Dollars)</th>
<th>Bankruptcy Trigger</th>
<th>Optimal Leverage</th>
<th>Principal Value (P)</th>
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<td>26</td>
<td>27.6</td>
</tr>
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<td>5.0</td>
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<td>35.75</td>
<td>37</td>
<td>40.1</td>
</tr>
<tr>
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<td>113.80</td>
<td>32.80</td>
<td>49</td>
<td>55.8</td>
</tr>
</tbody>
</table>

**Table 1:** Characteristic of Optimally Levered Firm

### Bankruptcy Level

- For long-term debt, \( V_B < P \).
- For short-term debt, \( V_B > P \).
Optimal Leverage (cont.)

<table>
<thead>
<tr>
<th>Maturity (Year)</th>
<th>Firm Value (Dollars)</th>
<th>Bankruptcy Trigger</th>
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<tr>
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</tr>
</tbody>
</table>

Table 1: Characteristic of Optimally Levered Firm

- Maximum firm value increases monotonically with debt maturity.
- Why would firms ever issue short term debt? (lower agency costs)
Debt Value and Debt Capacity

Figure 2: Debt value as a function of leverage.

- The maximum debt value is smaller for shorter maturity.
- Maximal debt value tends to occur at approximately the same leverage (about 80-85 present).
Debt Value and Debt Capacity (cont.)

Investment-Grade Bond and Junk Bond

- Debt value **falls** as $\sigma$ or $r$ increases when leverage is low.
- Debt value **increases** with $\sigma$ and with $r$ when leverage is (very) high and the bond is “junk.”
- As $r$ or $\sigma$ increases, the endogenous bankruptcy level $V_B$ is pushed down.
Debt Value and Debt Capacity (cont.)

Figure 3: Bond price and yield to maturity as functions of time to maturity.

- Low and intermediate leverages show “hump” market values of bonds.
- Very risky debt show a different price pattern.
The Term Structure of Credit Spreads

Figure 4: Credit spread as a function of debt maturity.

- For high leverage levels, spreads are high but decrease as issuance maturity $T$ increase beyond 1 year.
- For moderate-to-high leverage levels, spreads are distinctly humped.
- For low leverage level, credit spreads are low but increase with insurance maturity $T$. 
The Term Structure of Credit Spreads (cont.)

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<thead>
<tr>
<th></th>
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<tbody>
<tr>
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<td>$T = 5$</td>
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<tr>
<td></td>
<td>$T = 0.5$</td>
<td>$T = 5$</td>
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<tr>
<td>Base case</td>
<td>0.00</td>
<td>31.27</td>
</tr>
<tr>
<td>$\sigma = 20%, r = 7.5%, \alpha = 50%$</td>
<td>27.70</td>
<td>35.75</td>
</tr>
<tr>
<td>High volatility</td>
<td>0.00</td>
<td>86.74</td>
</tr>
<tr>
<td>$\sigma = 25%, r = 7.5%, \alpha = 50%$</td>
<td>26.98</td>
<td>33.72</td>
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<tr>
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<tr>
<td>$\sigma = 20%, r = 10%, \alpha = 50%$</td>
<td>26.15</td>
<td>33.03</td>
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<tr>
<td>Lower bankruptcy costs</td>
<td>0.00</td>
<td>11.53</td>
</tr>
<tr>
<td>$\sigma = 20%, r = 7.5%, \alpha = 50%$</td>
<td>20.94</td>
<td>31.83</td>
</tr>
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</table>

Table 2: Credit Spread and Endogenous Bankruptcy Trigger $V_B$ for Various Parameter Values.

The Effect of Endogenous versus Exogenous Bankruptcy on Credit Spreads

- Intermediate and long term debt always pay lower credit spreads in Panel A than in Panel B (higher $V_B \rightarrow$ lower debt value $\rightarrow$ higher cs).
## The Term Structure of Credit Spreads (cont.)

### Table 2: Credit Spread and Endogenous Bankruptcy Trigger $V_B$ for Various Parameter Values.

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Credit spreads fall when the default-free interest rate rises (unexpected results).
The Term Structure of Credit Spreads (cont.)

Table 2: Credit Spread and Endogenous Bankruptcy Trigger $V_B$ for Various Parameter Values.

<table>
<thead>
<tr>
<th>Exogenous Parameter Change before Debt is Issued</th>
<th>$T = 0.5$</th>
<th>$T = 5$</th>
<th>$T = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>0.00</td>
<td>31.27</td>
<td>111.10</td>
</tr>
<tr>
<td>$\sigma = 20%, r = 7.5%, \alpha = 50%$</td>
<td>27.70</td>
<td>35.75</td>
<td>35.32</td>
</tr>
<tr>
<td>High volatility</td>
<td>0.00</td>
<td>52.63</td>
<td>149.36</td>
</tr>
<tr>
<td>$\sigma = 25%, r = 7.5%, \alpha = 50%$</td>
<td>20.47</td>
<td>29.88</td>
<td>29.32</td>
</tr>
<tr>
<td>High risk-free rate</td>
<td>0.00</td>
<td>42.45</td>
<td>66.22</td>
</tr>
<tr>
<td>$\sigma = 20%, r = 10%, \alpha = 50%$</td>
<td>39.64</td>
<td>42.10</td>
<td>39.67</td>
</tr>
<tr>
<td>Lower bankruptcy costs</td>
<td>0.00</td>
<td>66.86</td>
<td>109.94</td>
</tr>
<tr>
<td>$\sigma = 20%, r = 7.5%, \alpha = 50%$</td>
<td>38.27</td>
<td>43.92</td>
<td>39.63</td>
</tr>
</tbody>
</table>

- A rise in the default-free rate will increase credit spreads of shorter term debt, but decrease spreads of long term debt.
The Term Structure of Credit Spreads (cont.)

<table>
<thead>
<tr>
<th>Panel C: Optimal $P$, $C$, and $V_B$</th>
<th>$T = 0.5$</th>
<th>$T = 5$</th>
<th>$T = 20$</th>
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</thead>
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<td>111.10</td>
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<tr>
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<td>35.75</td>
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</tbody>
</table>

Table 2: Credit Spread and Endogenous Bankruptcy Trigger $V_B$ for Various Parameter Values.

Bankruptcy Cost

- For intermediate term debt, lower bankruptcy costs lead to greater leverage, thereby increase yield spreads.
**The Duration and Convexity of Risky Debt**

Figure 5: Effective duration as a function of Macaulay duration.

- Macaulay duration
  \[ \text{Dur} = \frac{1 - e^{-RT}}{R}. \]  

- Effective duration: the percentage change in the value of newly issued bond with maturity \( T \) for a 1 percent change in \( r \).
The Duration and Convexity of Risky Debt (cont.)

**Figure 6**: Bond price as a function of interest rates.

- Riskless debt value is a **convex function** of the default-free interest $r$.
- As debt becomes increasingly risky, convexity is reduced and ultimately turns to concavity.
Bankruptcy Rates and Bond Ratings

**Figure 7:** Cumulative bankruptcy probabilities.

The cumulative probability of the firm going bankruptcy over the period \((0, s]\) is given by

\[
N\left(\frac{-b - \lambda s}{\sigma \sqrt{s}}\right) + e^{-2\lambda/\sigma^2} N\left(\frac{-b + \lambda s}{\sigma \sqrt{s}}\right),
\]

(30)

where \(\lambda = \mu - \delta - 0.5\sigma^2\).
Bankruptcy Rates and Bond Ratings (cont.)

**Bond Ratings**

- Probability of default versus credit spread.
- Credit spread seems more important variable to predict when market prices are unavailable.
Agency Effects

- After debt is issued, equity holders will wish to increase the riskiness of the firm’s activities.
- This is presumed to transfer value from debt to equity, create the asset substitution problem.
- This presumption follows from regarding equity as a call option on the firm’s asset, as indeed is the case when debt has no coupon, and taxes and bankruptcy costs are ignored.
- Barnea, Haugen, and Senbet (1980) suggest that shorter term debt may reduce shareholder incentives to increase risk.
Agency Effects (cont.)

- Equity in this paper is not precisely analogous to an ordinary call option.
  - Default may occur at any time.
  - $V_B$ varies with the risk of the firm’s activities.
  - The existence of tax benefit and potential bankruptcy cost.
Agency Effects (cont.)

Figure 8: Effects of an increase in risk $\sigma$ on bond and equity values.

- For either short or intermediate term debt, increasing risk will not benefit both bond and equity holders, except when bankruptcy is imminent.
Agency Effects (cont.)

Figure 8: Effects of an increase in risk $\sigma$ on bond and equity values.

- The incentives for increasing risk are much more pronounced for longer term debt.
At all maturity, the incentives for increasing risk become positive for both equity and bond holders as the bankruptcy level is approached.

Incentives to increase risk become positive for equity holders before they become positive for bond holders.

**Figure 8:** Effects of an increase in risk $\sigma$ on bond and equity values.
Bankruptcy Cost

- Firms with higher bankruptcy cost $\alpha$ will choose lower optimal amounts of debt.

- But they will lengthen the maturity of debt.

- This is due to the fact that longer maturity tends to reduce $V_B$ and thereby serves to postpone bankruptcy and its associated cost.
Agency Effects (cont.)

Payout

- The incremental firm value using long versus short term debt is significant reduced when $\delta$ falls.

- However, the incremental agency costs associated with asset substitution alone are lower for firms with lower payout rates.

- In this paper, they cannot predict whether optimal maturity should be shorten when $\delta$ falls.
Growth Prospects

- Firm with greater growth prospects typically have lower cash flows (as a fraction of asset value) available for payout to security holders, greater risk, and higher bankruptcy cost.

- Barclay and Smith’s (1995) empirical results show that firms with greater growth prospects do use shorter term debt.
The extent of conflict between equity holders and bond holders increase as tax rate and bankruptcy costs decline.
Agency Effects (cont.)

- In sum, these results suggest that the “general” asset substitution problem may have been overstated, except when debt is very long term, or when taxes and bankruptcy costs are minimal.

- The incompatibilities arise as bankruptcy is approached.

- On the very brink of bankruptcy, incentive compatibility is again restored.
This article develops a model of optimal leverage and risky corporate bond prices for arbitrary debt maturity.

Optimal leverage depends upon debt maturity and is markedly lower when the firm is financed by shorter term debt.

- The fact that longer term debt generates higher firm value poses the question of why firm issue short term debt.
Conclusion (cont.)

The choice of **debt maturity** represents a trade-off between tax advantages, bankruptcy costs, and agency costs.

- Risker firm should issue shorter term debt in addition to using less debt.
- A firm with higher bankruptcy costs will prefer longer term debt.
- The tax benefit of long term debt are significantly reduced when cash flows relative to asset value are small.
Conclusion (cont.)

- Risky corporate debt behaves very differently from default-free debt.
  - Effective duration may be shorter than Macaulay duration.

- Their techniques allow computation of the default probabilities, given the actual drift of the asset value process.
An EBIT-Based Model of Dynamic Capital Structure


They model the dynamics of the claim to earnings before interest and taxes (EBIT) as log-normal.

The implication is that all contingent claimants (equity, debt, and government, etc.) to future EBIT flows are treated in a consistent fashion.

The claim to EBIT flow should in practice be reasonably insensitive to changes in capital structure.

This feature makes their framework ideal for optimal dynamic capital structure.