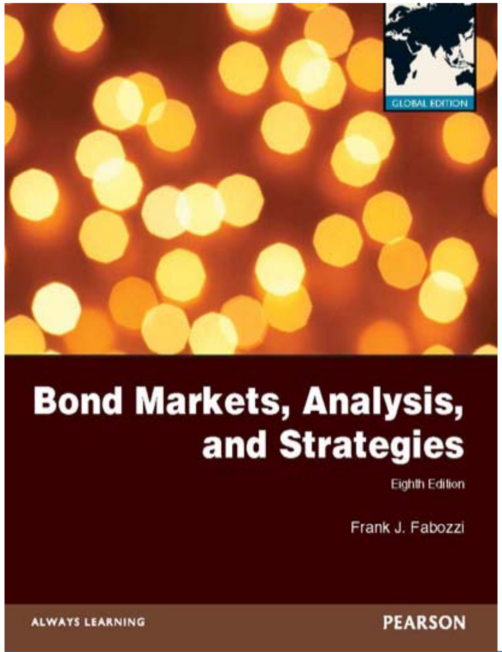


Global Edition

Chapter 17

Analysis of Bonds with Embedded Options

Drawbacks of Traditional Yield Spread Analysis

- Traditional analysis calculating the difference of the yield to maturity (or yield to call) of
  - the bond in question and
  - a comparable-maturity Treasury →obtained from the Treasury yield curve.

For example, consider two 8.8% coupon 25-year bonds:

Issue	Price	Yield to Maturity (%)
Treasury	$96.6133	9.15
Corporate	87.0798	10.24

- The yield spread for these two bonds= 10.24%-9.15%=109 bps
- The drawbacks are

(1) the yield for both bonds fails to take into consideration the term structure of interest rates,

(2) in the case of callable and/or putable bonds, expected interest rate volatility may alter the cash flow of a bond.

Static Spread: An Alternative to Yield Spread

- ❖ In traditional yield spread analysis,
  - ❖ compares the yield to maturity of a bond with that of a similar maturity on-the-run Treasury security.
- ❖ It makes little sense!
  - ❖ The cash flows of the corporate bond will not be the same as that of the benchmark Treasury.
- ❖ The proper way to compare non-Treasury bonds:
  - ❖ Compare with a portfolio of Treasury securities that have the same cash flow.
  - ❖ The corporate bond's value is equal to the PV of all the cash flows.

Static Spread: Discount the Cash flows with Zero Rates

- ❖ The corporate bond's value, given the cash flows are riskless, will equal the PV of the replicating portfolio of Treasury securities.
  - ❖ These cash flows are valued at the Treasury spot rates.
  - ❖ See next slide (Exhibit 17-1)
  - ○ The price would be $96.6133.
  - ○ The corporate bond's price is $87.0798
  - ○ Investors in fact require a yield premium for the risk associated with holding a corporate bond rather than a riskless package of Treasury securities.

Exhibit 17-1 Calculation of Price of a 25-Year 8.8% Coupon Bond Using Treasury Spot Rates

Period	Cash Flow	Treasury Spot Rate (%)	Present Value
1	4.4	7.00000	4.2512
2	4.4	7.04999	4.1055
3	4.4	7.09998	3.9628
4	4.4	7.12498	3.8251
5	4.4	7.13998	3.6922
6	4.4	7.16665	3.5622
....	....	....	....
46	4.4	10.10000	0.4563
47	4.4	10.30000	0.4154
48	4.4	10.50000	0.3774
49	4.4	10.60000	0.3503
50	104.4	10.80000	7.5278
Theoretical price			96.6134

Definitions of Static Spread

- Also called *the zero-volatility spread*,
- a measure of the spread over the entire Treasury spot rate curve if the bond is held to maturity.
- Not a spread over one point on the Treasury yield curve, l
  - like the traditional yield spread.
- Cash flows from the corporate bond discounted at the Treasury spot rate plus the static spread equal to the corporate bond's price.
- Exhibit 17-2 (next slide) illustrates the calculation of the static spread for a 25-year 8.8% coupon corporate bond.

Exhibit 17-2 Calculation of the Static Spread for a 25-Year 8.8% Coupon Corporate Bond

			Present Value if Spread Used Is:		
Period	Cash Flow	Treasury Spot Rate (%)	100 BP	110 BP	120 BP
1	4.4	7.00000	4.2308	4.2287	4.2267
2	4.4	7.04999	4.0661	4.0622	4.0583
3	4.4	7.09998	3.9059	3.9003	3.8947
4	4.4	7.12498	3.7521	3.7449	3.7377
5	4.4	7.13998	3.6043	3.5957	3.5871
....	....	....	....	....	....
46	4.4	10.10000	0.3668	0.3588	0.3511
47	4.4	10.30000	0.3323	0.3250	0.3179
48	4.4	10.50000	0.3006	0.2939	0.2873
49	4.4	10.60000	0.2778	0.2714	0.2652
50	104.4	10.80000	5.9416	5.8030	5.6677
Total present value			**88.5474**	**87.8029**	**87.0796**

Compare Yield Spread with Static Spread

- Exhibit 17-3 (Next slide) shows the static spread and the traditional yield spread for bonds with various maturities and prices.
- The shorter the maturity, the less the static spread will differ from the traditional yield spread.
- The magnitude of the difference between the yield spread and the static spread also depends on the shape of the yield curve.
  - The steeper the yield curve, the more the difference for a given coupon and maturity.
- Another reason for the small differences is that the corporate bond makes a bullet payment at maturity.

Exhibit 17-3 Comparison of Traditional Yield Spread and Static Spread for Various Bonds[a]

			Spread (basis points)		
Bond	Price	Yield to Maturity (%)	Traditional	Static	Difference
25-year 8.8% Coupon Bond					
Treasury	96.6133	9.15	—	—	—
A	88.5473	10.06	91	100	9
B	87.8031	10.15	100	110	10
C	87.0798	10.24	109	120	11
....	....	....	....	....	....
5-year 8.8% Coupon Bond					
Treasury	105.9555	7.36	—	—	—
J	101.7919	8.35	99	100	1
K	101.3867	8.45	109	110	1
L	100.9836	8.55	119	120	1

[a]Assumes Treasury spot rate curve given in Exhibit 17-1.

Callable Bonds and Their Investment Characteristics

- Two disadvantages of callable bonds to the bondholder:
  i. Expose bondholders to reinvestment risk
  ii. price appreciation potential for a callable bond in a declining interest-rate environment is limited
    - This phenomenon for a callable bond is referred to as *price compression*.
- If the investor receives sufficient potential compensation in the form of a higher potential yield, an investor would be willing to accept call risk.

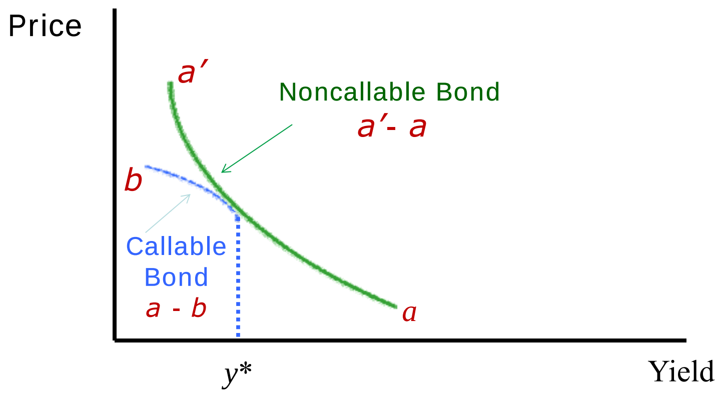
Traditional Valuation Methodology for Callable Bonds

- The practice has been to calculate a *yield to worst,*
  - the smallest of the yield to maturity and the yield to call for all possible call dates.
- The *yield to call* assumes that all cash flows can be reinvested at the computed yield until the assumed call date.
- The yield to call assumes that
  i. the investor will hold the bond to the assumed call date
  ii. the issuer will call the bond on that date.
- These underlying assumptions about the yield to call are unrealistic because they do not take into account how an investor will reinvest the proceeds if the issue is called.

Price-Yield Relationship for a Callable Bond

- Exhibit 17-4 (*next slide*) shows the price–yield relationship for both a noncallable bond and callable bond.
- The price–yield relationship for an option-free bond is convex.
  - See the convex curve *a–a′*
- The unusual shaped curve *a–b is the price–yield* relationship for the callable bond.
- The reason for the unusual shape for the callable bond:
  – When the prevailing market yield is higher than the coupon interest, it is unlikely that the issuer will call the bond.
  - If a callable bond is unlikely to be called, it will have the same convex price–yield relationship as a noncallable bond when yields are large enough

Exhibit 17-4 Price-Yield Relationship for a Noncallable and Callable Bond



Price-Yield Relationship for a Callable Bond (continued)

- As yields in the market decline, the likelihood that the issuer will call the bond increases.
- The exact yield level at which the issue is likely to be called may not be actually estimated: called it y^*.
- At yield levels below y^*, the price-yield relationship for the callable bond departs from that for the noncallable bond.
- For a range of yields below y^*, there is price compression
  - Limited price appreciation as yields decline.
- The portion of the callable bond price-yield relationship below y^* is said to be negatively convex.

Price-Yield Relationship for a Callable Bond (continued)

- *Negative convexity* implies price appreciation will be less than the price depreciation for a given change in yield.
- For a bond that is option-free:
  - positive convexity,
  - price appreciation will be greater than the price depreciation for a given change in yield.
- The price changes resulting from bonds exhibiting positive convexity and negative convexity are in Exhibit 17-5 (*see next slide*).
- It is important to understand that a bond can still trade above its call price even if it is highly likely to be called.

Exhibit 17-5 Price Volatility Implications of Positive and Negative Convexity

	Absolute Value of Percentage Price Change	
Change in Interest Rates	Positive Convexity	Negative Convexity
-100 basis points	*X%*	*Less than Y%*
+100 basis points	Less than *X%*	*Y%*

Components of a Bond with an Embedded Option (Callable Bonds)

- Decompose a bond with options into its component parts.
- In a callable bond:
  - Bondholder has sold the issuer a call option that allows the issuer to repurchase the bond from the first callable until the maturity date.
- The callable bond holder enters into two separate transactions:

i. buys a noncallable bond from the issuer
ii. sells the issuer a call option

- A callable bond is equal to the price of the two components parts:

  callable bond price = ***noncallable bond price – call option price***

- Graphically, this can be seen in Exhibit 17-6 (*see next slide*).
- The difference between the price of the noncallable bond and the callable bond at any given yield is the price of the embedded call option.

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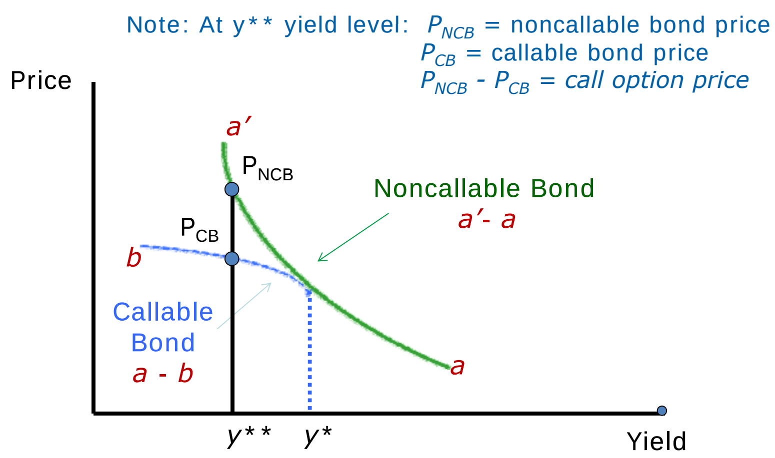
Exhibit 17-6 Decomposition of a Price of a Callable Bond



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Components of a Bond with an Embedded Option (Putable Bonds)

- The above logic can be applied to *putable bonds*.
- The bondholder has the right to sell the bond to the issuer at a designated price and time.
- A putable bond can be broken into two separate transactions.

  i. The investor buys a noncallable bond.

  ii. The investor buys an option from the issuer that allows the investor to sell the bond to the issuer.

- The price of a putable bond is then

  putable bond price = ***non-putable bond price*** + ***put option price***

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Valuation Model

- The bond valuation process requires that we use the theoretical spot rate to discount cash flows.
  - Equivalent to discounting at a series of forward rates.
- For an bond with embedded option, the valuation model considers how interest-rate volatility affects the bond value through its effects on the options.
- Three models can be used to account for the valuation effect of embedded options.

  i. Not a mortgage-backed security or asset-backed security and which can be exercised at more than once.
  ii. A bond with an embedded option where the option can be exercised only once.
  iii. A mortgage-backed security or certain types of asset-backed securities.

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Valuation of Option-Free Bonds

- PV of the cash flows discounted at the spot rates. Given the following hypothetical yield curve:

Maturity Years	Yield to Maturity (%)	Market Value
1	3.50	100
2	4.00	100
3	4.50	100

- Assuming annual-pay bonds. Using the bootstrapping methodology, the spot rates and the one-year forward rates can be obtained.

Years	Spot Rate (%)	One – Year Forward Rate
1	3.500	3.500
2	4.010	4.523
3	4.541	5.580

Valuation of Option-Free Bonds (continued)

- **EXAMPLE.** Consider an option-free bond with three years remaining to maturity and a coupon rate of 5.25%.
- The price of this bond can be calculated in one of two ways, both producing the same result.

i. The coupon payments can be discounted at the zero-coupon rates:

$$\frac{\$5.25}{1.035} + \frac{\$5.25}{(1.0401)^2} + \frac{\$100 + \$5.25}{(1.04541)^3} = \$102.075$$

ii. The second way is to discount by the one-year forward rates:

$$\frac{\$5.25}{1.035} + \frac{\$5.25}{(1.035)(1.04523)} + \frac{\$100 + \$5.25}{(1.035)(1.04523)(1.05580)} = \$102.075$$

Introducing Interest-Rate Volatility

- When we allow for embedded options, consideration must be given to interest-rate volatility.
- This can be done by introducing an *interest-rate tree*, or an *interest-rate lattice*.
- This tree is nothing more than a graphical depiction of the one-period future spot rates over time based on some assumed interest-rate model and interest-rate volatility.

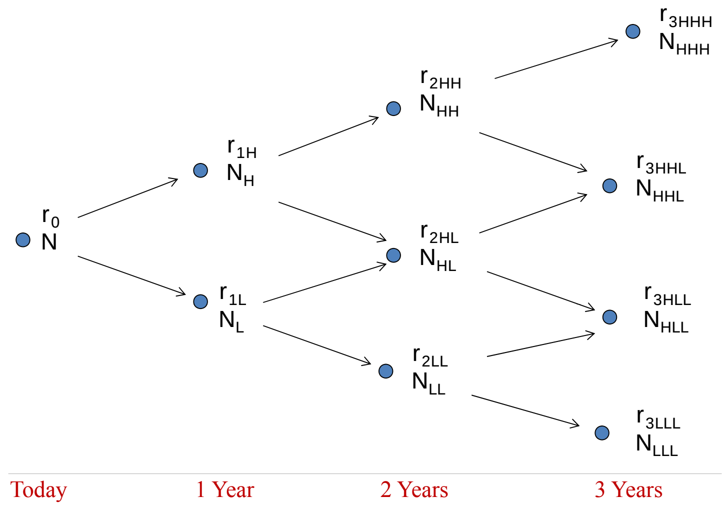
Interest-Rate Model

- It is a probabilistic description of how interest rates can change over the life of a derivatives.
  - Model the relationship between the level of short-term interest rates and interest-rate volatility
  - commonly use arbitrage-free models to describe how short-term interest rates can evolve over time.
- The interest-rate models based solely on movements in the short-term interest rate are referred to as one-factor models.
- More complex models would consider how more than one interest rate changes over time.

❖ Interest-Rate Tree

- Exhibit 17-7 (*next slide*) shows a basic type of interest-rate tree, a *binomial interest-rate tree*.
  - Referred to as the binomial model.
  - The interest rates can realize one of two possible rates in the next period.
- Valuation models that assume that interest rates can take on three possible rates in the next period are called *trinomial models*.
- More complex models exist that assume that more than three possible rates in the next period can be realized.

Exhibit 17-7 Three-Year Binomial Interest-Rate Tree



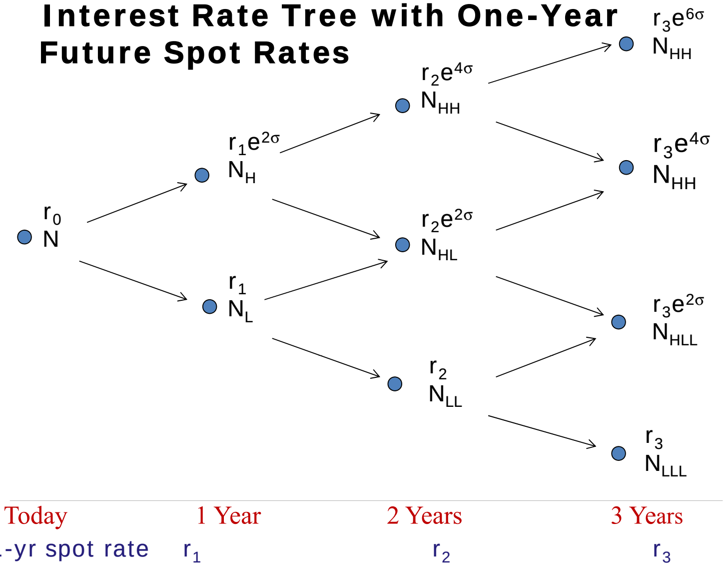
Interest-Rate Tree (continued)

- Each node (bold blue circle:) represents a time period that is equal to one year from the node to its left.
  - Labeled with an N, representing *node*,
- Subscript indicates the path that one-year future spot rates took to get to that node.
  - ***H*** represents the *higher* of the two future spot rates
  - ***L*** the *lower* of the two future spot rates .
  - Ex: node ***NHH*** means that to get to that node the following path for one-year rates occurred:
    - The one-year rate Realized the higher of the two future spot rates twice.

Interest-Rate Tree (continued)

- We can simplify the notation by letting $\boldsymbol{r_t}$ be the lower one-year future spot rate ***t*** years from now because all the other future spot rates ***t*** years from now depend on that rate.
- Exhibit 17-8 (*next slide*) shows the interest-rate tree using this simplified notation.
- Before we go on to show how to use this binomial interest-rate tree to value bonds, we first need to focus on
  i. what the volatility parameter (σ) represents
  ii. how to find the value of the bond at each node

Exhibit 17-8 Three-Year Binomial Interest Rate Tree with One-Year Future Spot Rates



Volatility and the Standard Deviation

- In the binomial model, it can be shown that the standard deviation of the one-year future spot rate is equal to $r_0\sigma$.
- The standard deviation is a statistical measure of volatility.
- Note that the volatility is measured relative to the current level of rates.
- **EXAMPLE.** If σ is 10% and the one-year rate (r_0) is 4%, what is the standard deviation of the one-year future spot rate ? What is if r_0 = 12%?

  $r_0\sigma$ = 4% × 10% = 0.4% or **40 basis points**

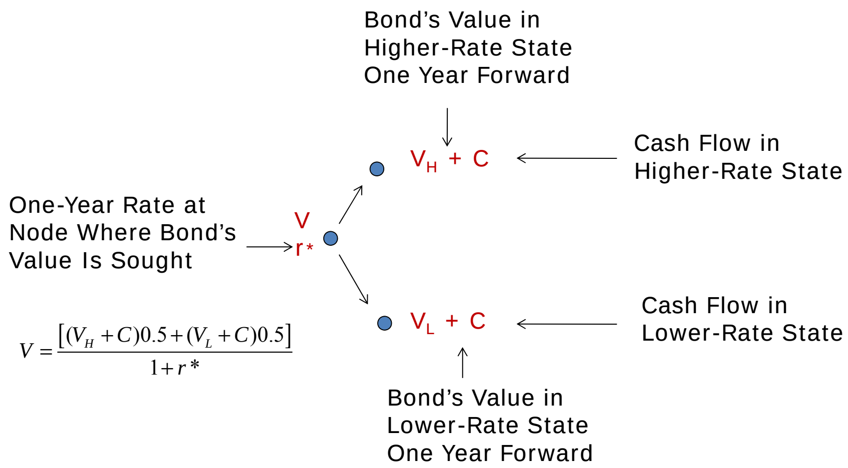
  $r_0\sigma$ = 12% × 10% = 1.2% or **120 basis points**

Determining the Bond Value at a Node

- In the binomial model, we find the bond value as illustrated in Exhibit 17-9 (*in next slide*).
- The future cash flow from present node will be either
  1. the bond's value if the short rate is the higher rate plus the coupon payment
  2. the bond's value if the short rate is the lower rate plus the coupon payment.

  The value at present node is the PV of the expected cash flows
  - Price with the backward induction method
  - the appropriate discount rate to use is the one-year future spot rate at the node.

Exhibit 17-9 Calculating a Value at a Node



$$V = \frac{[(V_H + C)0.5 + (V_L + C)0.5]}{1 + r^*}$$

Constructing the Binomial Interest-Rate Tree

- To construct the tree, we use current on-the-run yields and assume a volatility, σ.
- The root rate for the tree, r_0, is simply the current one-year rate.
- In the first year there are two possible one-year rates, the higher rate and the lower rate.
- What we want to find is the two future spot rates that will be consistent with the volatility assumption, the process that is assumed to generate the observed market value of the bond.
- The steps are described in Overheads 17-36, 17-37, and 17-38 and illustrated in Exhibits 17-10 and 17-11 (*see Overheads 17-39 and 17-40*).

Constructing the Binomial Interest-Rate Tree (Construct First Time Step)

→ **Step 1:** Arbitrarily Pick a r_1 : the lower one-year future spot rate one year from now, says 4.5%.

→ **Step 2:** Determine the higher one-year future spot rate with: $r_1(e^{2\sigma})$. This value is reported at node N_H.

→ **Step 3:** Compute the 2-year bond's value one year from now:

→ **3a.** The bond's value two years from now must be determined.

→ **3b.** Calculate the PV of the bond's value found in 3a using the higher rate: V_H.

→ **3c.** Calculate the PV of the bond's value found in 3a using the lower rate: V_L.

→ **3d.** Add the coupon to V_H and V_L to get the cash flow at N_H and N_L, respectively.

→ **3e.** Calculate PV of the two values using current spot r^*, so we can compute: $\frac{V_H + C}{1 + r_*}$ and $\frac{V_L + C}{1 + r_*}$.

❖ Constructing the Binomial Interest-Rate Tree (Construct First Time Step)

→ **Step 4:** Calculate the average PV of the two cash flows in step 3. This is the value at a node is $\frac{1}{2}\left[\frac{V_H + C}{1 + r_*} + \frac{V_L + C}{1 + r_*}\right]$.

→ **Step 5:** Compare the value in step 4 with the bond's market value.

- → If the two values are the same, r_1 is the rate we seek.
- → If the value found in step 4 is not equal to the market value of the bond, this means that the value r_1 is inconsistent with (1) the volatility assumption of 10%, (2) the observed market value of the bond.
- → Find a different value for r_1. and repeat the aforementioned process. [Note. If we get a value less than $100, then the value for r_1 is too large. Thus we try a lower value for r_1.]

→ Ex: when r_1 is 4.5% we get a value of $99.567 in step 4, which is less than the observed market value of $100 (See Exhibit 17-10).Therefore, 4.5% is too large and the five steps must be repeated, trying a lower value for r_1, says 4.074% (See Exhibit 17-11)

Exhibit 17-10 Finding the One-Year Forward Rates for Year 1 Using the Two-Year 4% On-the-Run: First Trial

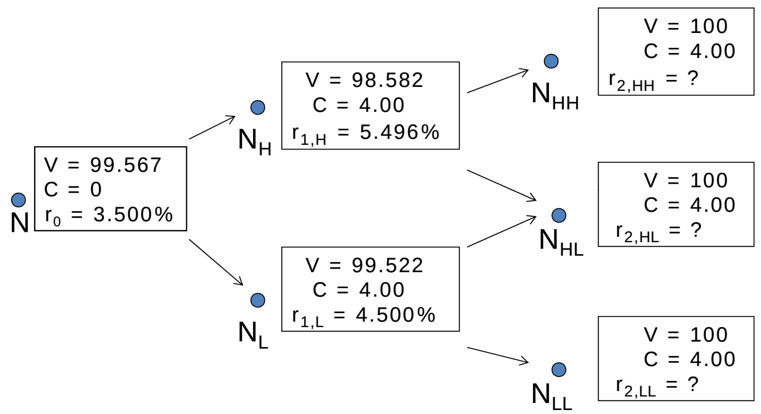
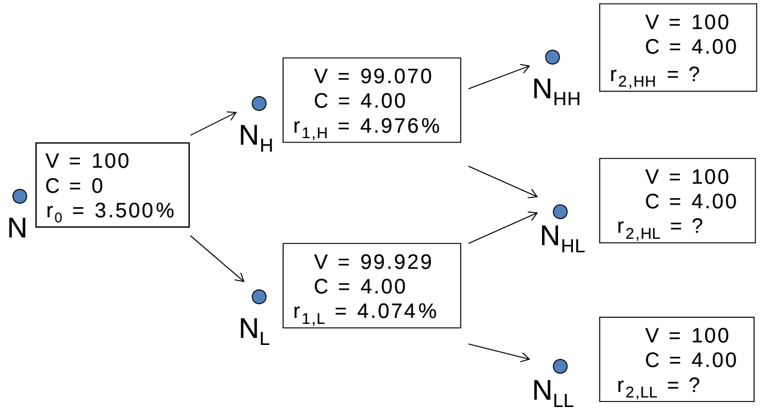


Exhibit 17-11 One-Year Forward Rates for Year 1 Using the Two-Year 4% On-the-Run Issue



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Constructing the Binomial Interest-Rate Tree (For Following Time Steps)

→ Next, we will use the three-year on-the-run issue to get r_2.

- → The same five steps are used iteratively process to find the one-year future spot rate two years from now.
- → Object is to find the value for r_2 that produce the value matching the observed market price.
- → The binomial interest-rate tree constructed is said to be an *arbitrage-free tree*. It is so named because it fairly prices the on-the-run issues.

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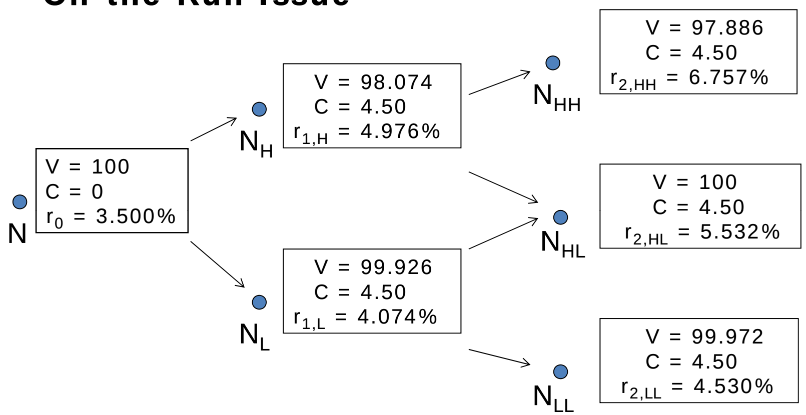
Exhibit 17-12 One-Year Future Spot Rates for Year 2 Using the Two-Year 4.5% On-the-Run Issue



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Application to Valuing an Option-Free Bond

- Consider a 5.25% corporate bond that has two years remaining to maturity and is option-free.
- Let the issuer's on-the-run yield curve is the one given earlier, and hence the appropriate binomial interest-rate tree is the one in Exhibit 17-12 (*the last slide*).
- Exhibit 17-13 (*next slide*) shows the backward induction procedure produces a bond value of $102.075.
- This value is identical to the bond value found earlier when we discounted at either the zero-coupon rates or the one-year forward rates.
- The valuation model is consistent with the standard valuation model for an option-free bond.

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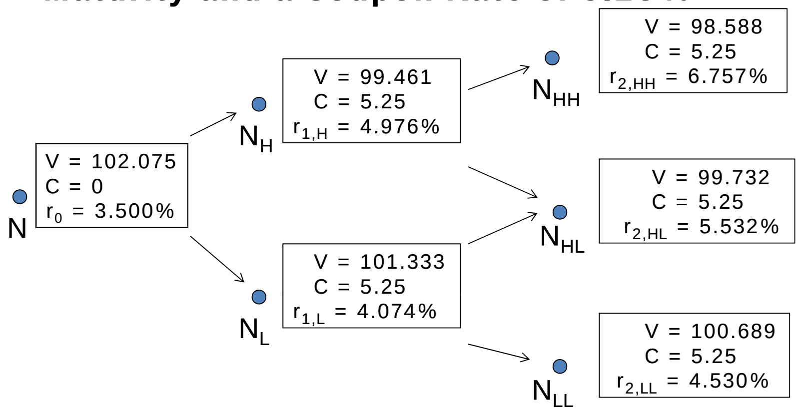
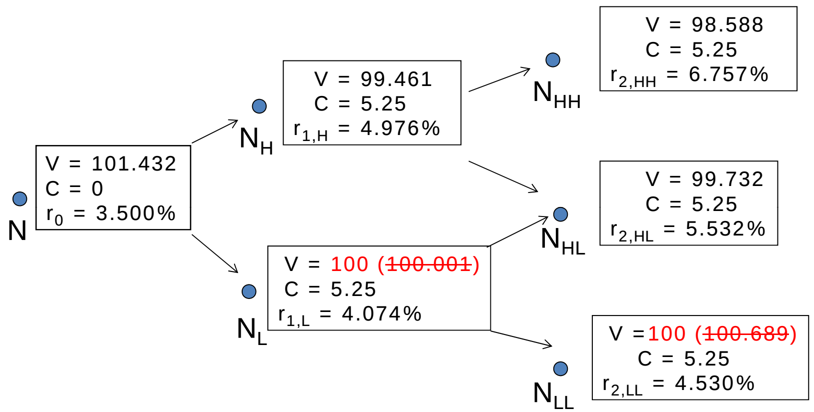
Exhibit 17-13 Valuing an Option-Free Corporate Bond with Three Years to Maturity and a Coupon Rate of 5.25%



Valuing a Callable Corporate Bond

- The valuation process for a callable corporate bond proceeds in the same fashion with one exception:
  - When the call option may be exercised by the issuer, the bond value at a node must be changed to reflect the lesser of its value if it is not called (i.e., the continuous value) and the call price.
- For example, consider a 5.25% corporate bond with three years remaining to maturity that is callable in one year at $100.
  - Exhibit 17-14 (*next slide*) shows the values at each node of the binomial interest-rate tree.

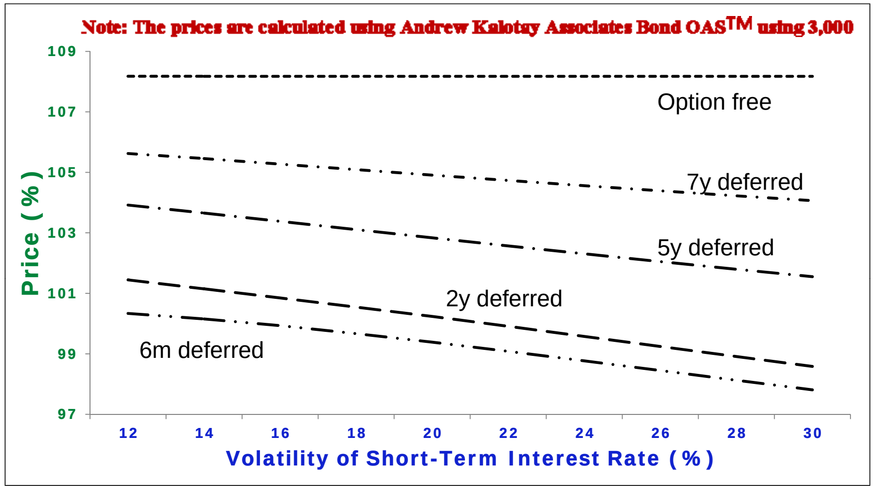
Exhibit 17-14 Valuing a Callable Corporate Bond with Three Years to Maturity and a Coupon Rate of 5.25%, and Callable in One Year at 100



Impact of Expected Interest Rate Volatility on Price

- Expected interest rate volatility is a key in the valuation of bonds with embedded options.
- Exhibit 17-15 (*next slide*) shows the price of four 5%, 10-year callable bonds with different deferred call structures (six months, two year, five years, and seven years) based on different expected volatility of short-term interest rates.
  1) The price of the option-free bond is the same regardless of the interest rate volatility.
     - No embedded option that is affected by interest rate volatility.
  2) For any given level of interest rate volatility, the longer the deferred call, the higher the price.
     - The value of the option-free bond has the highest price.
  3) The price of a callable bond moves inversely to the interest rate volatility.

Exhibit 17-15 Effect of Interest Rate Volatility and Years to Call on Prices of 5%, 10-Year Callable Bonds



Determining the Call Option Value (or Option Cost)

- The value of a callable bond is expressed as the difference between the values of a noncallable bond and the call option.

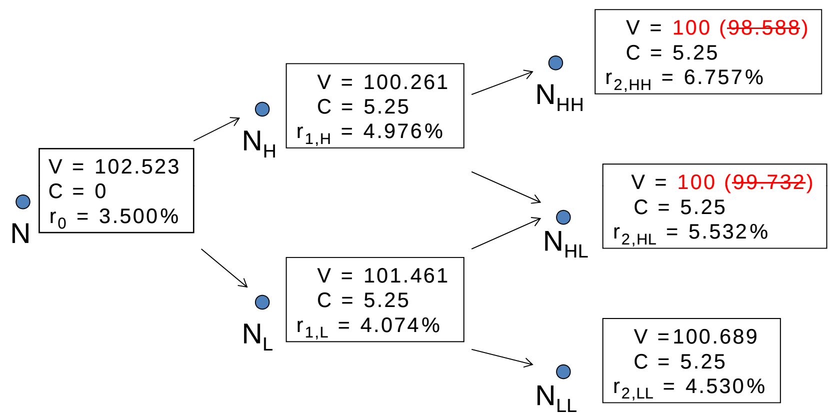
  value of a call option =
  value of a noncallable bond – value of a callable bond
- The value of a noncallable bond and the value of a callable bond can be determined:
  - The difference between the two values is therefore the value of the call option.
- In our previous illustration, the value of the noncallable bond is \$102.075 and the value of the callable bond is \$101.432, so the value of the call option is \$0.643.

Extension to Other Embedded Options

- This framework can be used to analyze other embedded options, such as put options, caps and floors on floating-rate notes
- Exhibit 17-16 (*next slide*) shows the binomial interest-rate tree for pricing putable bonds.
- Because the value of a non-putable bond can be expressed as the value of a putable bond minus the value of a put option on that bond, this means that

  value of a put option =
  value of a non-putable bond – value of a putable bond

Exhibit 17-16 Valuing a Putable Corporate Bond with Three Years to Maturity and a Coupon Rate of 5.25%, and Putable in One Year at 100



Remarks on Valuation Model

- **Incorporating Default Risk**
  - Can be extended to incorporate default risk.
  - Adjusting the probability of a payment default
  - Adjusting the cash flow that will be recovered when a default occurs.
- **Modeling Risk**
  - The risk that the output of the model is incorrect due to incorrect assumptions.
- **Implementation Challenge**
  - To transform the basic interest rate tree into a practical tool requires refinements.
    - The spacing of the time step in the tree must be much finer.
    - Introduce time-dependent node spacing could distort the term structure of volatility.

Option-Adjusted Spread

- The spread over the Treasury curve that make the theoretical price of an interest rate derivative equal to the market price.
- The reason that the resulting spread is referred to as "option-adjusted" is because the cash flows of the security whose value we seek are adjusted to reflect the embedded option.

Option-Adjusted Spread
(continued)

- **Translating OAS to Theoretical Value**
  - For a specified OAS, the valuation model can determine the theoretical value of the security that is consistent with that OAS.
  - As with the theoretical value, the OAS is affected by the assumed interest rate volatility.
  - The higher (lower) the expected interest rate volatility, the lower (higher) the OAS.
- **Determining the Option Value in Spread Terms**
  - The option value in spread terms is determined as follows:

option value (in bps) ***= static spread – OAS***

Effective Duration and Convexity

- Appropriate for estimating the modified durations for bonds with embedded options
- In general, the duration for any bond can be approximated as follows:

$$duration = \frac{P_{-} - P_{+}}{2(P_0)(dy)}$$

P_ = *price if yield is decreased by x bps*
P+ = *price if yield is increased by x bps*
P_0 = *initial price* (per $100 of par value)
Δy (or ***dy***) = *change in rate used to calculate price* (x basis points in decimal form)

Effective Duration and Convexity
(continued)

- When the approximate duration formula is applied to a bond with an embedded option, the bond prices at the higher and lower yield levels are evaluated from the valuation model.
- Duration calculated in this way is called *effective duration* or *option-adjusted duration*.
- The differences between modified duration and effective duration are summarized in Exhibit 17-17 (*next slide*).
- The standard convexity measure may be inappropriate for a bond with embedded options because it does not consider the effect of a change in interest rates on the bond's cash flow.

Exhibit 17-17 Modified Duration Versus Effective Duration