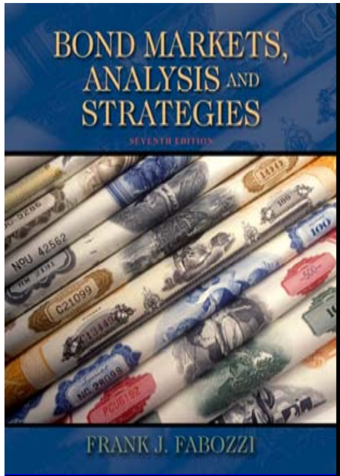

Chapter 2

Pricing of Bonds

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Learning Objectives

- time value of money
- Calculate the price of a bond
  - estimate the expected cash flows
  - determine the yield to discount
- Bond price changes reversely with the yield

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Learning Objectives
(continued)

- The relationship between price and yield of an option-free bond is convex
- The relationship between coupon rate, time to maturity, bond yield, and bond price
- the pricing of floating-rate and inverse-floating-rate securities
- accrued interest, dirty price, and clean (quoted) price

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Review of Time Value

❖ **Future Value (FV)**

The FV (P_n) of any sum of money invested today is:

$$P_n = P_0(1+r)^n$$

n = *number of periods*

P_n = *future value* **n** *periods from now* (in dollars)

P_0 = *original principal* (in dollars)

r = *interest rate per period* (in decimal form)

$(1+r)^n$ represents the future value of \$1 invested today for n periods at a compounding rate of r

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Review of Time Value (FV)

❖More than one time per year,

both the interest rate and the number of periods must be adjusted

r → *annual interest rate* ÷ number of interest payout per year

n → *number of times payout per year* × *number of years*

FV increases with the number of compounding per year:

reflects the greater opportunity for reinvesting the interest paid.

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Time Value of Money

- Periodic compounding

(If interest is compounded *m* times per annum)

$$FV = PV\left(1+\frac{r}{m}\right)^{nm}$$

- Continuous compounding

$$FV = PVe^{rn}$$

$$\lim_{t\to\infty}(1+\frac{1}{t})^{t} = e \rightarrow \lim_{m\to\infty}(1+\frac{r}{m})^{nm} = \lim_{m\to\infty}(1+\frac{1}{m/r})^{\frac{m}{r}rn} = e^{rn}$$

- Simple compounding

Review of Time Value (Annuity)

❖ **FV for Ordinary Annuity**

❖**Annuity:**

Investing the same amount of money periodically.

❖ **Ordinary annuity:**

First investment occurs one period from now.

The equation for the future value of an ordinary annuity (P_n) is:

$$P_n = A\left[\frac{(1+r)^n - 1}{r}\right]$$

A = *the amount of the annuity* (in dollars).

r = *annual interest rate* ÷ *number of times interest paid per year*

n = *number of times interest paid per year times number of years*

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Review of Time Value (Annuity)

❖ **Example of Future Value of an Ordinary Annuity Using Annual Interest:**

If ***A*** = \$2,000,000, ***r*** = 0.08, and ***n*** = 15, then P_n = **?**

$$P_n = A\left[\frac{(1+r)^n - 1}{r}\right]$$

$$P_n = \$2,000,000\left[\frac{(1+0.08)^{15} - 1}{0.08}\right]$$

P_n = \$2,000,000 [27.152125] = **\$54,304.250**

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Review of Time Value (Annuity)

❖ **Example of Future Value of an Ordinary Annuity Using Semiannual Interest:**

If A = \$2,000,000/2 = \$1,000,000, r = 0.08/2 = 0.04, and n = 15(2) = 30, then P_n = ?

$$P_n = A\left[\frac{(1+r)^n - 1}{r}\right]$$

$$P_n = \$1,000,000\left[\frac{(1+0.04)^{30} - 1}{0.04}\right]$$

P_n = \$1,000,000 [56.085] = **\$56,085,000> \$54,304.250**

Review of Time Value (PV)

❖ **Present Value(PV)**

PV is the FV process in reverse. We have:

$$P_n = \left[\frac{1}{(1+r)^n}\right]$$

r *= annual interest rate ÷ number of times interest paid per year*
n *= number of times interest paid per year times number of years*

➢ PV decreases when interest rate r tends to be higher or the time to payment date n tends to be longer.

Review of Time Value (PV)

❖ **PV of a Series of FVs**

➢ Calculate the PV of each FV by discounting

➢ Then these present values are added together to obtain the present value of the entire series of future values.

Review of Time Value (Annuity)

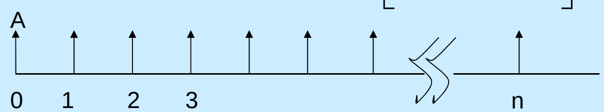
❖ **Present Value of an Annuity due**

When the first payment is immediate, the annuity is called an annuity due.
The PV of an annuity due is:

$$C\frac{1-(1+r)^{-n}}{r}(1+r)$$

The PV of an ordinary annuity is

$$PV = A\left[\frac{1 - 1/(1+r)^n}{r}\right]$$



Review of Time Value (continued)

❖ **Example of Present Value of an Ordinary Annuity (*PV*) Using Annual Interest:**

If A = \$100, r = 0.09, and n = 8, then PV = ?

$$PV = A\left[\frac{1-1/(1+r)^n}{r}\right]$$

$$PV = \$100\left[\frac{1-1/(1+0.09)^8}{0.09}\right]$$

PV = \$100 [5.534811] = **\$553.48**

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Pricing a Bond

❖ Evaluating a financial instrument requires an estimate of:

i. the expected cash flows
ii. the appropriate required yield that reflects the yield
   i. for financial instruments with comparable risk
   ii. alternative investments

❖ The cash flows for a bond that the issuer cannot retire prior to its stated maturity date consist of

i. periodic coupon payments to the maturity date
ii. the par (maturity) value at maturity

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Value of a Coupon Bond

The bond price P can be computed using the following formula:

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} + \frac{M_t}{(1+r)^n}$$

Assume the coupon is paid semiannually.

P = *price* (in dollars)
n = *number of periods (number of years times 2)*
t = *time period when the payment is to be received*
C = *semiannual coupon payment (in dollars)*
r = *periodic interest rate (required annual yield divided by 2)*
M = *maturity value*

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An Example of Pricing a Coupon Bond

PV of coupons:

❖Consider a 20-year 10% coupon bond with a par value of \$1,000 and a required yield of 11%.

❖Given C = 0.1(\$1,000) / 2 = **\$50**, n = 2(20) = **40** and r = 0.11 / 2 = **0.055**, PV of the coupon payments is:

$$C\left[\frac{1-1/(1+r)^n}{r}\right]$$

$$= \$50\left[\frac{1-1/(1+0.055)^{40}}{0.055}\right]$$

= \$50 [16.046131] = **\$802.31**

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An Example of Pricing a Coupon Bond

PV of par value.

❖The PV of the par or maturity value of $1,000 is:

$$\left[\frac{M}{(1+r)^n}\right]=\left[\frac{\$1,000}{(1+0.055)^{40}}\right]= \$\mathbf{117.46}$$

Continuing the computation from the previous slide:

The price of the bond (***P***) =

PV(coupon payments)+ PV(maturity value) =

$802.31 + $117.46 = **$919.77**.

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The Value of a Zero Coupon Bond

For **zero-coupon bonds**, interest is the difference between the maturity value and the purchase price.

$$P=\frac{M_t}{(1+r)^n}$$

P = *price* (in dollars)

M = *maturity value*

r = *periodic interest rate* (required annual yield divided by 2)

n = *number of periods* (number of years times 2)

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An Example of Pricing a Zero-Coupon Bond

❖ **Zero-Coupon Bond Example**

Consider the price of a zero-coupon bond (***P***) that matures 15 years from now, if the maturity value is $1,000 and the required yield is 9.4%. Given ***M*** = **$1,000**, ***r*** = 0.094 / 2 = **0.047**, and ***n*** = 2(15) = **30**, what is ***P*** ?

$$P=\frac{M_t}{(1+r)^n}=\frac{\$1,000}{(1+0.047)^{30}}=\$252.12$$

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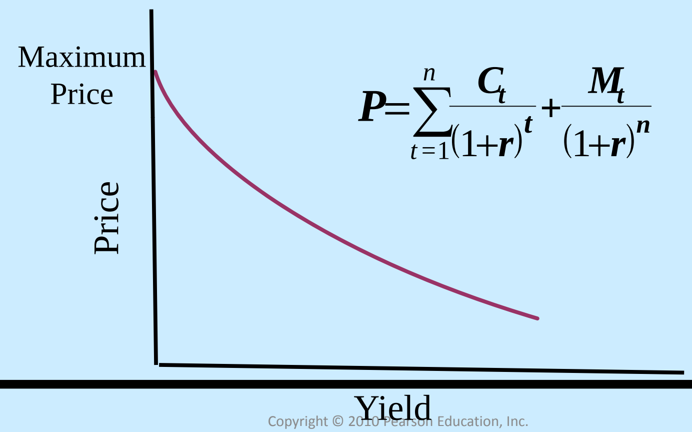
Price-Yield Relationship

➢ Price changes in the opposite direction from the change in the required yield



$$P=\sum_{t=1}^{n}\frac{C_t}{(1+r)^t}+\frac{M_t}{(1+r)^n}$$

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Exhibit 2-1

Price-Yield Relationship for a 20-Year 10% Coupon Bond

Yield	Price ($)	Yield	Price ($)	Yield	Price ($)
0.055	1,541.76	0.085	1,143.08	0.125	817.70
0.060	1,462.30	0.090	1,092.01	0.130	787.82
0.065	1,388.65	0.095	1,044.41	0.135	759.75
0.050	1,627.57	**0.100**	**1,000.00**	0.140	733.37
0.070	1,320.33	0.110	$919.77	0.145	708.53
0.075	1,256.89	0.115	883.50	0.150	685.14
0.080	1,197.93	0.120	849.54	0.155	663.08

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Relationship Between Coupon Rate, Required Yield, and Price

- When yields rise above the coupon rate,
  - the price of the bond falls so that an investor buying the bond can realizes capital appreciation.
  - The appreciation represents a form of interest to a new investor to compensate for coupon rate <required yield.
- When a bond sells below its par value, it is said to be *selling at a discount.*
- A bond whose price is above its par value is said to be *selling at a premium.*

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Relationship Between Bond Price and Time

- For a bond selling at par value,
  - coupon rate= required yield.
  - Bond price remains par as the bond moves toward the maturity date.
- The price of a bond will *not* remain constant for a bond selling at a premium or a discount.
- Exhibit 2-3 (Next slide) shows the time path of a 20-year 10% coupon bond selling at a discount and the same bond selling at a premium as it approaches maturity.
  - ✓ The discount bond increases in price as it approaches maturity, assuming that the required yield does not change.
  - ✓ For a premium bond, the opposite occurs.
  - ✓ For both bonds, the price will equal par value at the maturity date.

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Exhibit 2-3

Time Path for the Price of a 20-Year 10% Bond Selling at a Discount and Premium as It Approaches Maturity

Year	Price of Discount Bond Selling to Yield 12%	Price of Premium Bond Selling to Yield 7.8%
20.0	$ 849.54	$1,221.00
16.0	859.16	1,199.14
12.0	874.50	1,169.45
10.0	885.30	1,150.83
8.0	898.94	1,129.13
4.0	937.90	1,074.37
0.0	1,000.00	1,000.00

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Reasons for the Change in the Price of a Bond

The price of a bond can change for three reasons:

i. change in the required yield owing to changes in the credit quality of the issuer

ii. change in the price of the bond selling at a premium or a discount, without any change in the required yield, simply because the bond is moving toward maturity

iii. change in the required yield owing to a change in the yield on comparable bonds (i.e., a change in the yield required by the market)

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Complications

❖ The framework for pricing a bond assumes the following:

1) the next coupon payment is exactly six months away
2) the cash flows are known
3) the appropriate required yield can be determined
4) one rate is used to discount all cash flows

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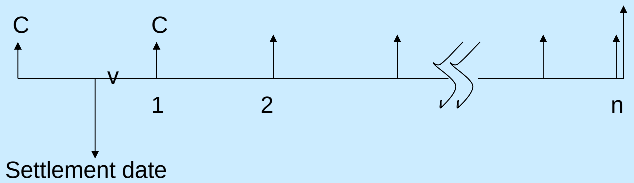
Settle between Coupon Payment Dates

Purchases a bond whose next coupon payment is due in less than six months:

$$P=\sum_{t=1}^{n}\frac{C}{(1+r)^{v}(1+r)^{t-1}}+\frac{M}{(1+r)^{v}(1+r)^{t-1}}$$

where v = (*days between settlement and next coupon*) divided by (*days in six-month period*)



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Complications

❖ **Cash Flows May Not Be Known**

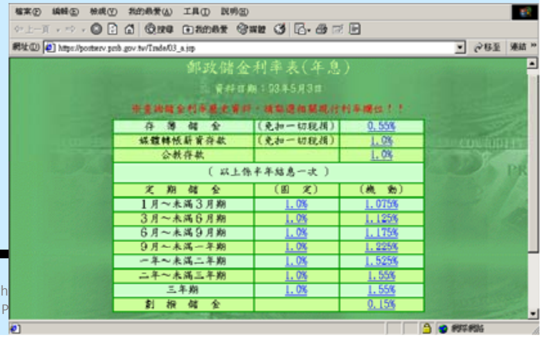
- callable bond
- floating rate bond

❖ **Determining the Appropriate Required Yield**

- Treasury yields as benchmark.
- Decompose the required yield for a bond into its component parts.

❖ **One Discount Rate Applicable to All Cash Flows**

- discount with yield rate
- A bond can be viewed as a package of zero-coupon bonds,
  - each cash flow is discounted with zero rate



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Pricing Floating-Rate and Inverse-Floating-Rate Securities

➢ The cash flow for either a *floating-rate* or an *inverse-floating-rate* security depends on the future reference rate

Price of a Floater

➢ The coupon rate of a floating-rate security (or floater) = reference rate + spread (or margin).

➢ The price of a floater depends on

i.the spread over the reference rate

ii.any restrictions imposed on the resetting of the coupon rate

i.Ex: a floater may have a maximum coupon rate called a cap or a minimum coupon rate called a floor.

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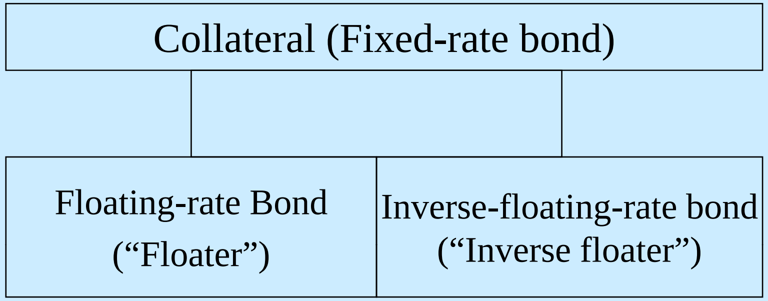
Pricing Inverse-Floating-Rate Securities

❖ **Price of an Inverse-Floater**

➢ Created from a fixed-rate security → called collateral

✓From the collateral two bonds are created: a floater and an inverse floater.



➢The price of an inverse floater equals the collateral’s price minus the floater’s price.

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Price Quotes and Accrued Interest

Price Quotes

❖A bond selling at par is quoted as 100, meaning 100% of its par value.

❖A bond selling at a discount will be selling for less than 100.

❖A bond selling at a premium will be selling for more than 100.

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Price Quotes and Accrued Interest (continued)

❖Traders quoting the bond price as a percentage of par value.

- Exhibit 2-5 in next slide shows how a quote price is converted into a dollar price.

❖When an investor purchases a bond between coupon payments,

  ❖ the investor must compensate the seller the coupon interest earned from the last coupon date to the settlement date of the bond.

- This amount is called accrued interest.
- For corporate and municipal bonds, accrued interest is based on a 360-day year, with each month having 30 days.

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Exhibit 2-5

Price Quotes Converted into a Dollar Price

(1) Price Quote	(2) Converted to a Decimal [= 1)/100]	(3) Par Value	(4) Dollar Price [= (2) x (3)]
80 1/8	0.8012500	10,000	8,012.50
76 5/32	0.7615625	1,000,000	761,562.50
86 11/64	0.8617188	100,000	86,171.88
100	1.0000000	50,000	50,000.00
109	1.0900000	1,000	1,090.00
103 3/4	1.0375000	100,000	103,750.00
105 3/8	1.0537500	25,000	26,343.75

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Price Quotes and Accrued Interest
(continued)

- The amount that the buyer pays the seller is the agreed-upon price plus accrued interest.
  - Called *full price* or *dirty price*.
- The bond price without accrued interest is called the *clean price*.
- The exceptions are bonds that are in default.
- Such bonds are said to be quoted flat, that is, without accrued interest.

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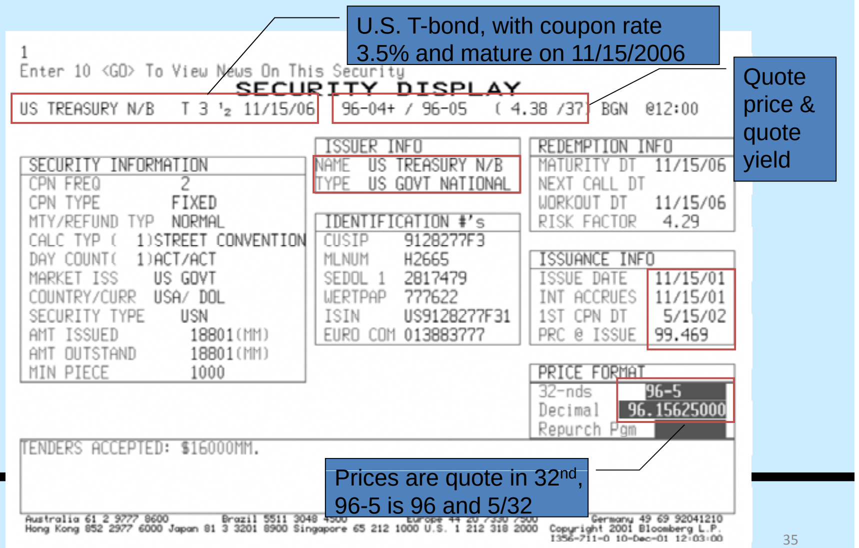
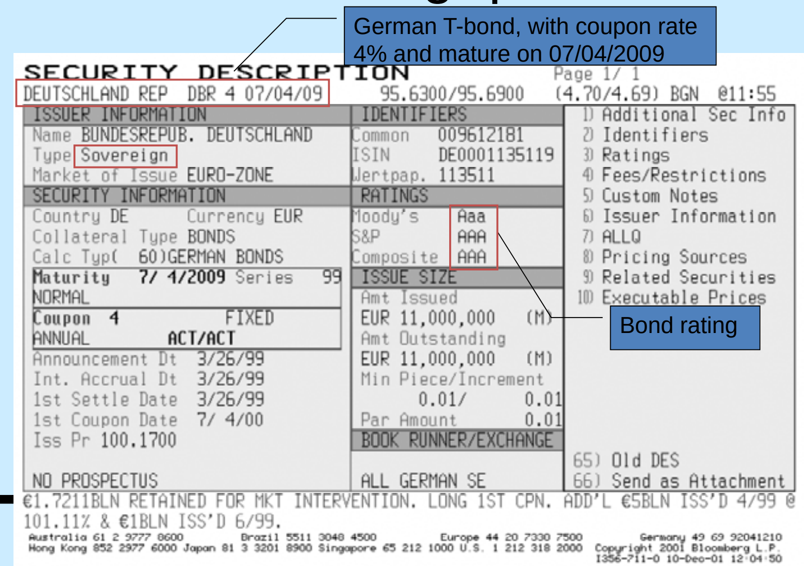
Bloomberg quotes



Bloomberg quotes



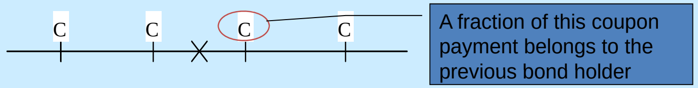
Price quotes

- bond prices are quoted as a percentage of par value
- Examples

(1) Price Quote	(2) Converted to a Decimal [= (1)/100]	(3) Par Value	(4) Dollar Price [= (2) × (3)]
97	0.9700000	$ 10,000	$ 9,700.00
$85\frac{1}{2}$	0.8550000	100,000	85,500.00
$90\frac{1}{4}$	0.9025000	5,000	4,512.50
$80\frac{1}{8}$	0.8012500	10,000	8,012.50
$76\frac{5}{32}$	0.7615625	1,000,000	761,562.50
$86\frac{11}{64}$	0.8617188	100,000	86,171.88
100	1.0000000	50,000	50,000.00
109	1.0900000	1,000	1,090.00
$103\frac{3}{4}$	1.0375000	100,000	103,750.00
$105\frac{3}{8}$	1.0537500	25,000	26,343.75
$103\frac{19}{32}$	1.0359375	1,000,000	1,035,937.50

Price quotes

- Bond quoted price
  - Clean price
    - = gross price –accrued interest
    - = the sum of PV of the future cash flows – accrued interest
    - Gross price is the price that bond buyer must pay
    - Gross/Dirty/full price= clean price + accrued interest
  - Accrued Interest
    - When an investor purchases a bond between coupon payments
    - the interest payment previous bondholder should receive
    - the quoted prices do not include accrued interest
    - buyers must pay the quote bond price + accrued interest



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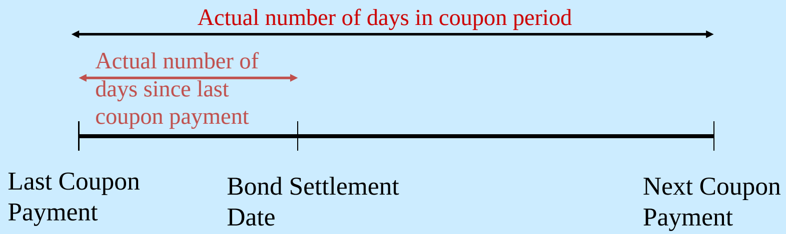
$$Accrued\ Interest = C \times \frac{days\ since\ last\ coupon\ payment}{days\ between\ coupon\ payments}$$

Accrued Interest Calculation (Semi-annual coupon payment)

$$accrued\ interest = \frac{int}{2} \times \frac{actual\ number\ of\ days\ since\ last\ coupon\ payment}{actual\ number\ of\ days\ in\ coupon\ period}$$



Bloomberg quotes

- Bond quoted price

Example 1.4 An investor buys on 12/10/01 a given amount of the US Treasury bond with coupon 3.5% and maturity 11/15/2006. The current clean price is 96.15625. Hence the market value of \$1 million face value of this bond is equal to 96.15625% × \$1 million = \$961,562.5. The accrued interest period is equal to 26 days. Indeed, this is the number of calendar days between the settlement date (12/11/2001) and the last coupon payment date (11/15/2001). Hence the accrued interest is equal to the last coupon payment (1.75, because the coupon frequency is semiannual) times 26 divided by the number of calendar days between the next coupon payment date (05/15/2002) and the last coupon payment date (11/15/2001). In this case, the accrued interest is equal to 1.75 × (26/181) = 0.25138. The gross price is then 96.40763. The investor will pay \$964,076.3 (96.40763% × \$1 million) to buy this bond.

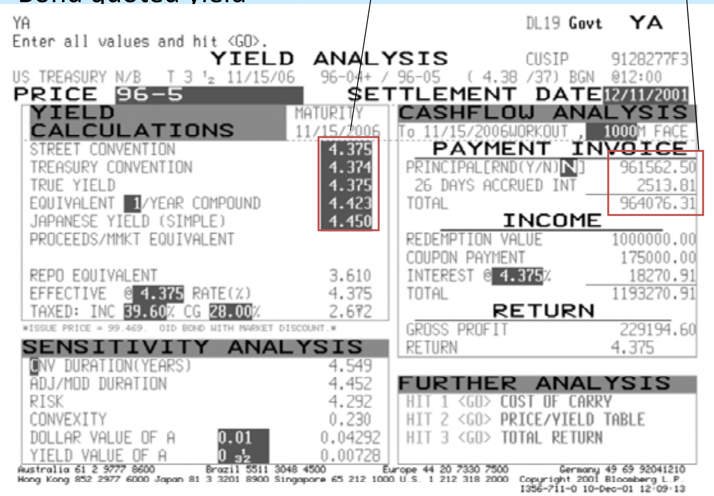
Note that the *clean price* of a bond is equal to the gross price on each coupon payment date and that US bond prices are commonly quoted in /32ths.

Bloomberg quotes

Gross, clean price and accrued interest

A variety of yields is quoted

- Bond quoted yield



Bloomberg quotes

Bond yield expressed as the spreads over some benchmark yields

- Bond quoted spread