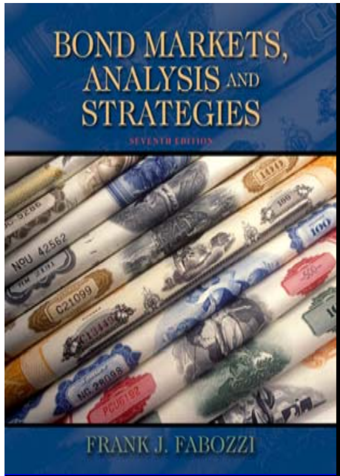


Chapter 3

Measuring Yield

Learning Objectives

After reading this chapter, you will understand

- Calculate the yield on any investment portfolio
- Calculate the current yield, yield to maturity, yield to call, yield to put, and cash flow yield
- Calculate the discount margin for a floating-rate security
- three potential sources of a bond's return

Learning Objectives

- what reinvestment risk is
- the limitations of conventional yield measures
- Calculate the total return for a bond
- why the total return is superior to conventional yield measures
- how to use horizon analysis to assess the potential return

Computing the Yield or Internal Rate of Return (IRR) on any Investment

- The yield on any investment is the interest rate that will make the PV of the cash flows from the investment equal to the *price* (or cost) of the investment.
- The yield on any investment, ***y***, is the interest rate that satisfies the equation.

$$P = \frac{CF_1}{1+y} + \frac{CF_2}{(1+y)^2} + \frac{CF_3}{(1+y)^3} + \dots + \frac{CF_N}{(1+y)^N}$$

where ***P*** = *price of the investment*, ***CF*** = *cash flow in year* ***t*** =1,2,3,… ***N***, ***y*** = *internal rate of return*, ***N*** = *number of years.*

Number of Payments

- ❑ The yield computed is the yield for the period.
  - ❑ Ex::
  - ❑ cash flows are semiannual, the yield is a semiannual yield.
  - ❑ cash flows are monthly, the yield is a monthly yield.
- ❑ To compute the *simple* annual interest rate, the yield for the period is multiplied by the number of periods in the year.

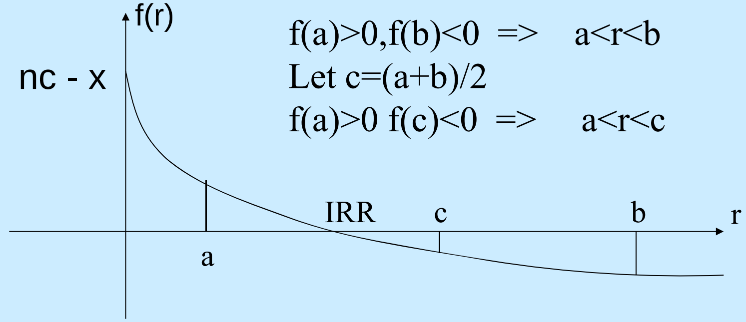
❑ Solving IRR Using Bisection Procedure.

- $f(r) = CF_1 \times (1+r)^{-1} + CF_2 \times (1+r)^{-2} + \ldots + CF_N \times (1+r)^{-N} - X$
- Solve $f(r)=0$

f(a)>0,f(b)<0 => a<r<b

Let c=(a+b)/2

f(a)>0 f(c)<0 => a<r<c



Multiple IRR

- Multiple IRR arise when there is more than one sign reversal in the cash flow pattern, and it is also possible to have no IRR.
- Evaluating real investment, IRR rule breaks down when there are multiple IRR or no IRR.

Class Exercise (Excel)

12	Time	CF	
13	0	-1000	
14	1	800	
15	2	1000	
16	3	1300	
17	4	-2200	=IRR(B13:B17,0.1)
18		7%	
19		37%	=IRR(B13:B17,0.2)
20	Multiple IRR		
21			

Special Case: Investment with Only One Future Cash Flow

- When the case where there is only one future cash flow we can use the below equation:

$$y = \left[\frac{CF_n}{P}\right]^{\frac{1}{n}} - 1$$

Computing the Effective Annual Yield

- Related with frequency of compounding

  effective annual yield = (1 + *periodic interest rate*)^{m} – 1

  where ***m*** is the frequency of payments per year.

- **Ex: quarterly payment with rate 8%:**
  - *periodic interest rate* is 0.08 / 4 = 0.02.

effective annual yield = (1.02)^{4} – 1 = 1.0824 – 1 = 0.0824 or **8.24%**

Determine Periodic Rate with Effective Annual Yield

- Determine the *periodic interest rate* that will produce a given effective annual interest:
- i.e:

  (1+ *periodic interest rate*)^m=1+ *effective annual yield*

Periodic rate is solved as:

periodic interest rate = (1 + *effective annual yield*)$^{1/m}$ – 1

- **Ex: find** *periodic quarterly interest rate* that produces an *effective annual yield* of 12%, then we have:

periodic interest rate = (1.12)$^{1/4}$ – 1 = 1.0287 – 1 = 0.0287 or **2.87%**

Annual rate=2.87%*4=11.48%

Conventional Yield Measures

- Bond yield measures commonly quoted by dealers and used by portfolio managers are:

1) Current Yield
2) Yield To Maturity
3) Yield To Call
4) Yield To Put
5) Yield To Worst
6) Cash Flow Yield
7) Yield (Internal Rate of Return) for a Portfolio
8) Yield Spread Measures for Floating-Rate Securities

Conventional Yield Measures
Current Yield

Current yield relates the *annual coupon interest* to the market *price*:

current yield = *annual dollar coupon interest* / *price*

Conventional Yield Measures
Yield To Maturity

The *yield to maturity* is the rate to make the PV of the cash flows equal to the price (i.e. IRR).

- For a semiannual pay bond, the *yield to maturity* is found by first computing the *periodic interest rate*, ***y***, which satisfies the relationship:

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \dots + \frac{C}{(1+y)^n} + \frac{M}{(1+y)^n}$$

where ***P*** = *price of the bond*, ***C*** = *semiannual coupon interest* (in dollars), ***M*** = *maturity value* (in dollars), and ***n*** = *number of periods* (number of years multiplied by 2).

Yield to maturity =2y.

Computed on the aforementioned basis called:

bond equivalent yield

i.e.: Annualize yield with semiannual compounding

Conventional Yield Measures
Yield To Maturity (continued)

For zero coupon bonds:

$$y = \left[\frac{M}{P}\right]^{1/n} - 1$$

where ***M*** = *maturity value* (in dollars), ***P*** = *price of the bond*, and ***n*** = *number of periods* (number of years multiplied by 2).

- The *yield-to-maturity* calculation takes into account (i) the current coupon income, (ii) any capital gain or loss realized by holding the bond to maturity, and (iii) the timing of the cash flows.

Bond selling at	Relationship
Premium	Coupon rate>current yield>yield to maturity
Par	Coupon rate=current yield=yield to maturity
Discount	Coupon rate<current yield<yield to maturity

Conventional Yield Measures
Yield To Call

The *call price* is the price at which the bond may be called .

- There is a call schedule that specifies a *call price* for each call date.
- The *yield to call* assumes that the issuer will call the bond at some assumed call date n* and the prespecified *call price M** :
- The *yield to call* can be expressed as follows:

$$P = \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \dots + \frac{C}{(1+y)^{n^*}} + \frac{M^*}{(1+y)^{n^*}}$$

where ***M**** = *call price* (in dollars) and ***n**** = *number of periods* until the assumed call date (number of years times 2)

- For a semiannual pay bond, doubling the *periodic interest rate* (***y***) gives the *yield to call on a bond-equivalent basis.*

Conventional Yield Measures
Yield To Put

A putable bond allows the bondholder to force the issuer to buy back the bond at the prespecified “put price”.

- When an issue is putable, a *yield to put* is calculated.
- The *yield to put* is the interest rate that makes PV of the cash flows to the assumed put date plus the *put price* equal to the bond’s price.
- The formula for the *yield to put* is the same as for the *yield to call*, but ***M*** * is now defined as the *put price* and ***n**** is the number of periods until the assumed put date.
- The procedure is the same as calculating the *yield to maturity* and the *yield to call*.

Conventional Yield Measures
Yield to Worst/ Cash flow Yield

Yield To Worst

- A practice in the industry is for an investor to calculate the *yield to maturity*, the yield to every possible call date, and the yield to every possible put date.
- The minimum of all of these yields is called the *yield to worst*.

Cash Flow Yield

- **Amortizing securities** involve cash flows that include interest plus principal repayment
- Prepayment: borrower may repay exceed the principal amounts.
- Thus cash flow each period consists of : (i) coupon interest, (ii) scheduled principal repayment, and (iii) prepayments.
- *Cash flow yield* is the interest rate that will make the present value of the amortizing securities equal to the market price.

See Ch. 11.

Conventional Yield Measures
Yield for a Portfolio

- The *yield for a portfolio of bonds* is not simply the average or weighted average of the *yield to maturity* of the individual bond issues in the portfolio.
- It is computed by determining the cash flows for the portfolio and determining the interest rate that will make the PV of the cash flows equal to the market value of the portfolio.

Bond	Coupon Rate	Maturity	Par Value	Price
A	7%	5	10000000	9209000
B	10.50%	7	20000000	20000000
C	6%	3	30000000	28050000
			Sum	57259000

Period	Bond A	Bond B	Bond C	Portfolio Cash flow
1	350000	1050000	900000	2300000
2	350000	1050000	900000	2300000
3	350000	1050000	900000	2300000
4	350000	1050000	900000	2300000
5	350000	1050000	900000	2300000
6	350000	1050000	30900000	32300000
7	350000	1050000		1400000
8	350000	1050000		1400000
9	350000	1050000		1400000
10	10350000	1050000		11400000
11		1050000		1050000
12		1050000		1050000
13		1050000		1050000
14		21050000		21050000

4.77% makes the PV of portfolio cash flows equal to the market price of the portfolio→ annual 9.54%

Conventional Yield Measures

Yield Spread Measures for Floating-Rate Securities

- The *coupon* rate for a floating-rate security changes periodically based on the coupon reset formula.
- This formula consists of the *reference rate* and the *quoted margin*.
- Since the future value for the *reference rate* is unknown, it is not possible to determine the cash flows.
  - *yield to maturity* cannot be computed.

Conventional Yield Measures

Yield Spread Measures for Floating-Rate Securities

Several conventional measures used as *margin* or *spread measures* cited by market participants for floaters:

- *spread for life* (or *simple margin*),
- *adjusted simple margin*,
- *adjusted total margin*,
- *discount margin*: the most popular
  - Estimates the average margin over the *reference rate* that the investor can expect to earn over the life of the security.
- To evaluate the discount margin:
  1. Determine the cash flow given the reference rate doesn't change
  2. Select a margin (spread)
  3. Discount the cash flow by the reference rate+ the margin (picked in step 2)
  4. Discount margin=margin in step 2 when PV of cash flows in step 3 =market price.
  5. Repeat step 2~4 to find the discount margin

Exhibit 3-1 Calculation of the Discount Margin for a Floating-Rate Security

Floating-rate security
Maturity: six years
Coupon rate: reference rate + 80 bps
Reset every six months

			PV of Cash Flow at Assumed Annual Margin (bps)				
Period	Reference Rate	Cash Flow[a]	80	84	88	96	100
1	10%	5.4	5.1233	5.1224	5.1214	5.1195	5.1185
2	10	5.4	4.8609	4.8590	4.8572	4.8535	4.8516
3	10	5.4	4.6118	4.6092	4.6066	4.6013	4.5987
4	10	5.4	4.3755	4.3722	4.3689	4.3623	4.3590
5	10	5.4	4.1514	4.1474	4.1435	4.1356	4.1317
6	10	5.4	3.9387	3.9342	3.9297	3.9208	3.9163
7	10	5.4	3.7369	3.7319	3.7270	3.7171	3.7122
8	10	5.4	3.5454	3.5401	3.5347	3.5240	3.5186
9	10	5.4	3.3638	3.3580	3.3523	3.3409	3.3352
10	10	5.4	3.1914	3.1854	3.1794	3.1673	3.1613
11	10	5.4	3.0279	3.0216	3.0153	3.0028	2.9965
12	10	105.4	56.0729	55.9454	55.8182	55.5647	55.4385
	Present Value =		100.0000	99.8269	99.6541	99.3098	99.1381

[a]For periods 1–11: cash flow = 100 (reference rate + assumed margin)(0.5); for period 12: cash flow = 100 (reference rate + assumed margin)(0.5) + 100.

Potential Sources of a Bond's Dollar Return

- An investor who purchases a bond can expect to receive a dollar return from one or more of these sources:

i. the periodic coupon interest payments
ii. any capital gain/loss when the bond matures, is called, or is sold
iii. interest income generated from reinvestment of the periodic cash flows → reinvestment income
   – For non-amortizing securities, it's called interest-on-interest component

- The *current yield* considers only the coupon interest payments.
- The *yield to maturity*, *yield to call*, and *cash flow yield* take all the three components into account.
  - the payments are assumed to be **reinvested at the same computed yield**

Determining the Interest-On-Interest Dollar Return (Focus on non-amortizing securities)

- The *interest-on-interest* component can represent a substantial portion of a bond's potential return. The *coupon interest* plus *interest on interest* can be found by using the following equation:

$$\textit{coupon interest} + \textit{interest on interest} = \boldsymbol{C}\left[\frac{(1+\boldsymbol{r})^{\boldsymbol{n}}-1}{\boldsymbol{r}}\right]$$

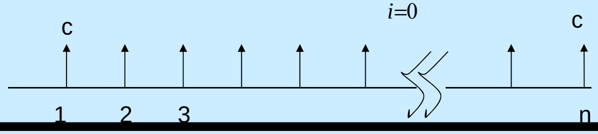
where

C is the *coupon interest*

r is the *semiannual reinvestment rate*

n is the *number of periods*

$$\sum_{i=0}^{n-1} C(1+r)^i = C\frac{(1+r)^n-1}{r}$$



Determining the Interest-On-Interest Dollar Return (continued)

- The total dollar amount of coupon interest is

total coupon interest = ***nC***

where ***C*** is the *coupon interest*, ***r*** is the *semiannual reinvestment rate*, and ***n*** is the *number of periods*.

The *interest-on-interest* component is the difference between

the *coupon interest* plus *interest on interest*

the *total dollar coupon interest*

$$\textit{interest on interest} = \boldsymbol{C}\left[\frac{(1+\boldsymbol{r})^{\boldsymbol{n}}-1}{\boldsymbol{r}}\right]-\boldsymbol{nC}$$

Determining the Interest-On-Interest Dollar Return Example

- Assume that the *coupon interest* (***C***) is $35/Period, the *semiannual reinvestment rate* (***r***) is 5%, and the *number of periods* (***n***) is 30. What is the *interest-on-interest*?

Using our equation for *interest on interest* and inserting in our given values we get:

$$\textit{interest on interest} = \boldsymbol{C}\left[\frac{(1+\boldsymbol{r})^{\boldsymbol{n}}-1}{\boldsymbol{r}}\right]-\boldsymbol{nC} =$$

$$35\left[\frac{(1+0.05)^{30}-1}{0.05}\right]-30(35)=2325.36-1050=\mathbf{\$1,275.36}$$

Yield To Maturity and Reinvestment Risk

- The investor realizes the *yield to maturity* only if
  - bond is held to maturity
  - coupon payments can be reinvested at the computed *yield to maturity*.
- *Reinvestment risk* is the risk that future reinvestment rates will be less than the *yield to maturity* at the time the bond is purchased.
- Two characteristics of a bond that determine the importance of the *interest-on- interest* component and therefore the degree of reinvestment risk:
  - ***maturity***
  - ***coupon***.

Yield To Maturity and Reinvestment Risk (continued)

- For a given yield to maturity and a given *coupon rate*, the longer the maturity, the more dependent the bond's total dollar return is on the *interest-on-interest* component in order to realize the *yield to maturity*.
- For a given maturity and a given *yield to maturity*, higher coupon rates will make the bond's total dollar return more dependent on the reinvestment of the coupon payments in order to produce the *yield to maturity*.

Cash Flow Yield and Reinvestment Risk

- For amortizing securities, reinvestment risk is even greater than for non-amortizing securities.
  - since investor must now reinvest the periodic principal repayments in addition to the periodic coupon interest payments.
- For non-amortizing securities, the borrower can accelerate the periodic principal repayment, in particular, a borrower will tend to prepay when interest rates decline.
- If a borrower prepays when interest rates decline, the investor faces greater reinvestment risk because he must reinvest the prepaid principal at a lower interest rate.

Total Return

- The *yield to maturity* is a **promised yield** because at the time of purchase an investor is promised a yield, if both of the following conditions are satisfied:
  (i) the bond is held to maturity
  (ii) all coupon interest payments are reinvested at the *yield to maturity*
- The **total return** is a measure of yield that incorporates an explicit assumption about the *reinvestment rate*.
- The *yield-to-call (put)* measure is subject to the same problems as the *yield to maturity* because it assumes that the:
  (i) bond will be held until the first call date
  (ii) coupon interest payments will be reinvested at the *yield to call*

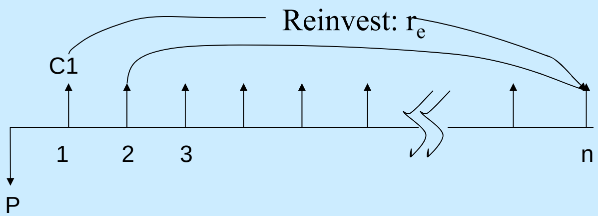
Computing the Total Return for a Bond

.

i. First, compute the total future dollars that will result from investing in a bond assuming a particular *reinvestment rate*.

ii. The *total return* is then computed as the interest rate that will make the initial investment in the bond grow to the computed total future dollars.

See next slide:



- The FV of investment in n period is $FV = P(1+y)^n$
- Let the reinvestment rates r_e, the FV of per cash income is

$$C_1 \times (1+r_e)^{n-1} + C_2 \times (1+r_e)^{n-2} + \ldots + C_{n-1} \times (1+r_e) \ + C_n \rightarrow \text{Value is given}$$

- We define Total Return (y) is

$$P(1+y)^n = C \times (1+r_e)^{n-1} + C \times (1+r_e)^{n-2} + \ldots + C \times (1+r_e) \ + C$$

Applications of the Total Return Horizon Analysis

- **Horizon analysis** refers to using *total return* to assess performance over some investment horizon.
- **Horizon return** refers to when a *total return* is calculated over an investment horizon.
- An application to the *total return* measure is that it requires the portfolio manager to formulate assumptions about
  - reinvestment rates
  - future yields

to calculate the horizon return.

An Analysis Example

- Consider a investment with 5-year investment horizon

Bond	coupon(%)	Maturity	YTM
A	5	3	9
B	6	20	8.6
C	11	15	9.2
D	8	5	8

- Bond C has higher YTM, but may suffer capital loss when selling the bonds at 5 year.
- Bond A has shorter maturity, but it may suffer the reinvestment risk.
- Estimate by total return with assumptions
  – reinvestment rate
  – future yields

Example for Calculating Total Return

- Three-year investment horizon
- Buy 20-year 8% bond for 828.4
- Reinvestment rate 6%
- At the end of third year, 17-year bond's yield =7%
- Step 1: coupon +reinvestment

$$40\left[\frac{(1.03)^6 - 1}{0.03}\right] = 258.74$$

Example for Calculating Total Return (continued)

- Step 2: Calculate the price to sell the bond

$$1098.51 = 40 \times (1+3.5\%)^{-1} + 40 \times (1+3.5\%)^{-2} + ... + 1040 \times (1+3.5\%)^{-34}$$

- Step 3: Total return:

$$\left[\frac{1098.51 + 258.74}{828.4}\right]^{6} - 1 = 8.58\%$$

- Annualized total return =17.16%

Calculating Yield Changes

- The **absolute yield change** (or **absolute rate change**) is measured in bps and is the absolute value of the difference between the two yields as given by

$$\textit{absolute yield change} = |\textit{initial yield} - \textit{new yield}| \times 100 \text{ (in bps)}$$

- The percentage change is computed as the natural logarithm of the ratio of the change in yield as shown by

$$\textit{percentage change yield} = 100 \times \ln(\textit{new yield} / \textit{initial yield}) \text{ (in \%)}$$

where *ln* in the natural logarithm.

Ex: Let M1: 4.45%, M2:5.11%, we have

|4.45%-5.11%|*100 =66bps

ln(5.11%/4.45%)*100=13.83%